Chapter 2. Baseband processing

Chapter 2. BB Processing Outline

- General Aspects
- Bit-level Processing
- Modulation
- Pulse-shape Filtering

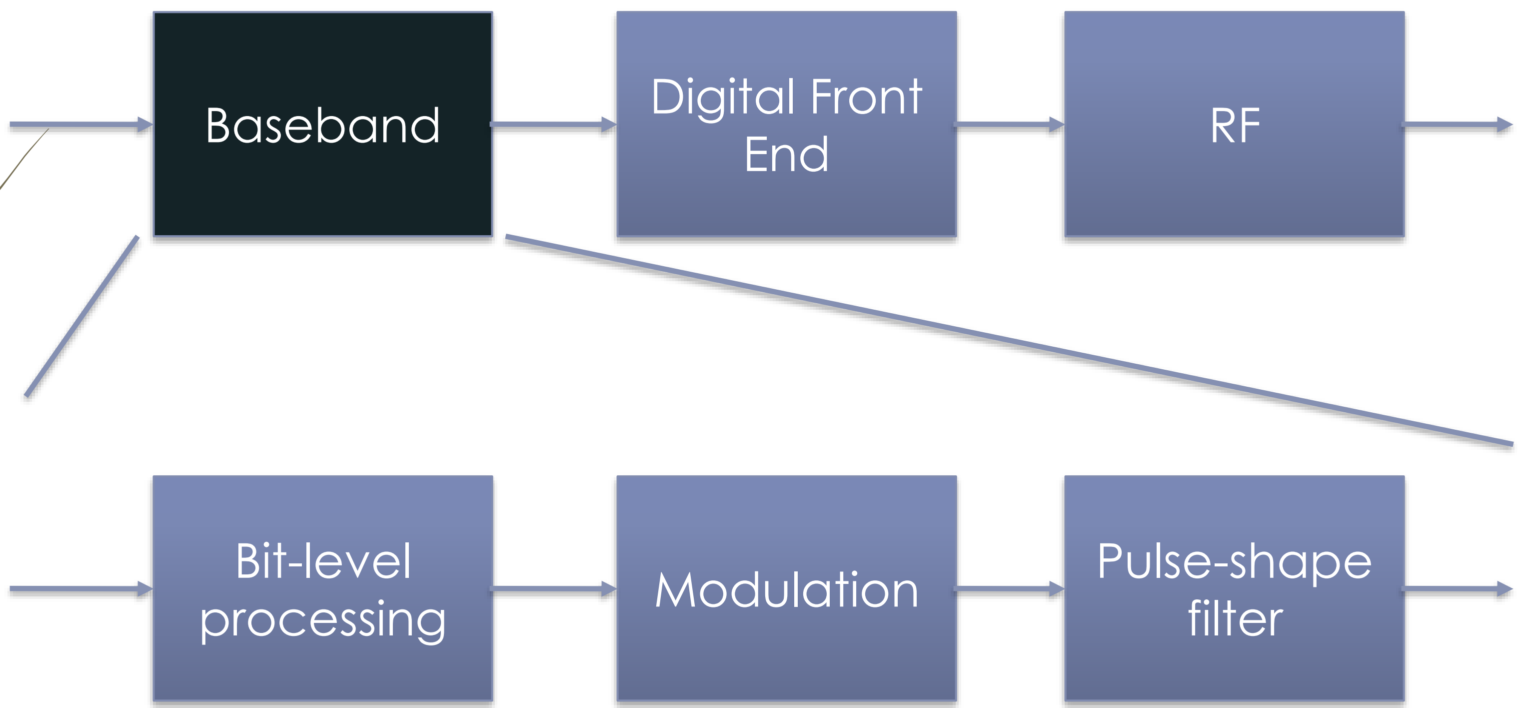
General Aspects

- Basic digital communication system
  - Tx

  - Rx

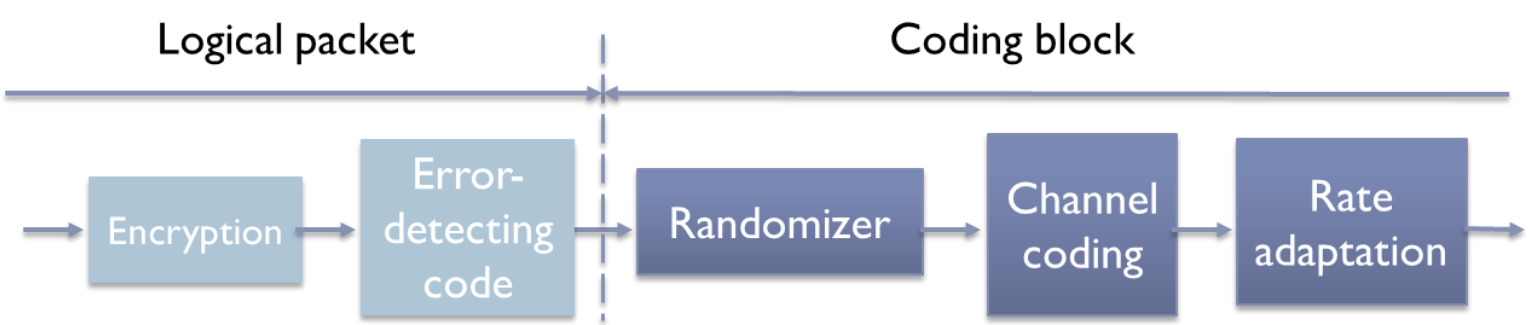
General Aspects

- Baseband processing - Tx



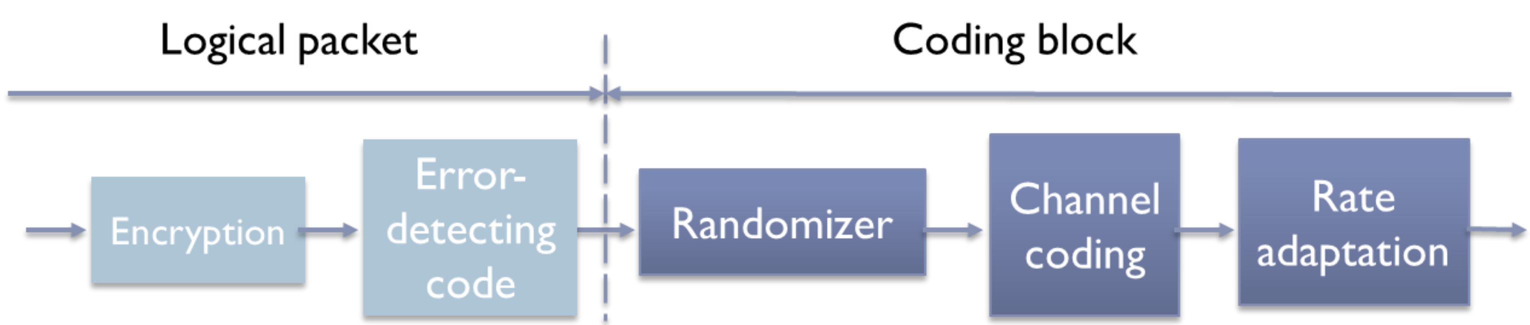
Bit-level Processing

- Encryption
  - Functionally, is performed in the upper layer if the stack
  - Practically, is performed at the physical layer, because of the easy implementation
- Error-detecting code
  - Functionally, is performed in the upper layer if the stack
  - Inserting the CRC in the packet
- Bit distribution uniformization (randomization)
  - Randomizer



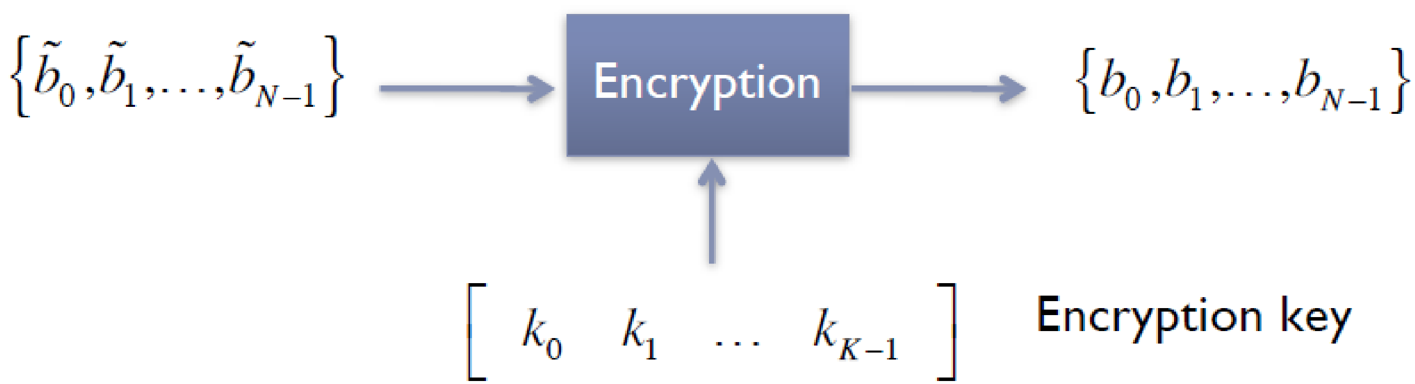
Bit-level Processing (2)

- Robustness for propagation over radio channels
  - Introducing redundancy bits
- Rate adaptation
  - Puncturing operation



Encryption

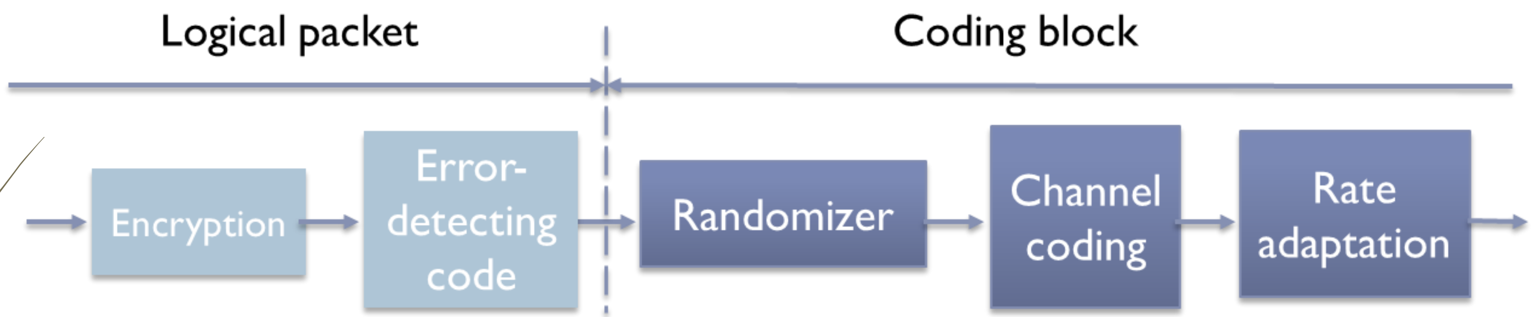
- **Goal:** Apply a change to the data packet, using a key known only by the transmitter and the receiver
  - The key is handled and managed at logical level
- The processing is done at packet level



Encryption (2)

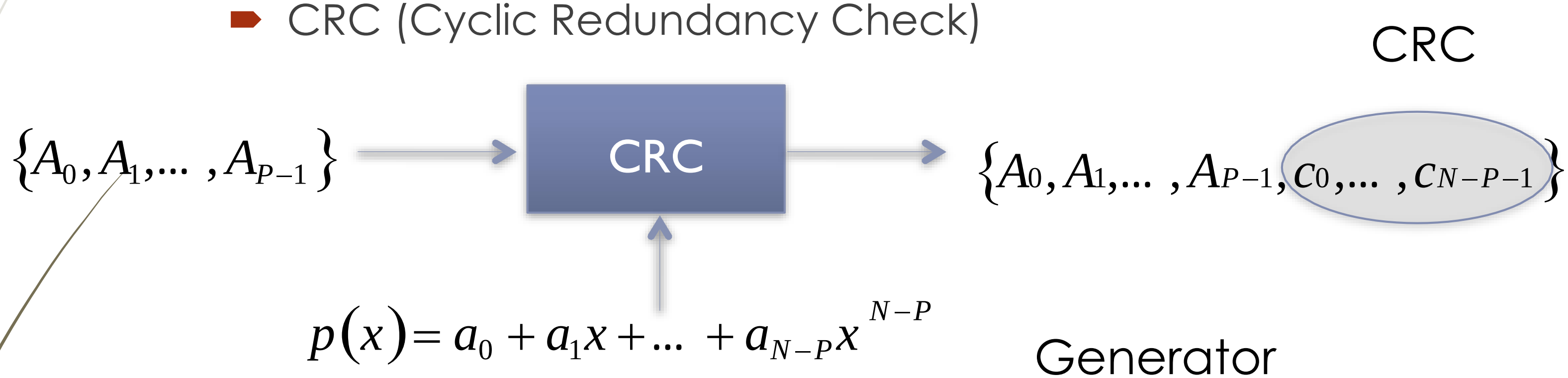
- **Encryption algorithms:**
  - **DES – Data Encryption Standard** (IBM-1976)
    - Key on 56 bits
    - Too short: it was broken in 22h15m
  - **AES – Advanced Encryption Standard** (1998)
    - Key on 128 | 192 | 256 bits
    - Block of 128 bits
- Any encryption algorithm uses an encryption key
  - CCM (Counter with CBC-MAC) – cipher of 128 bit
- The operation is done at logical packet level
- Optionally: CRC (Cyclic Redundancy Check)

Bit-level Processing



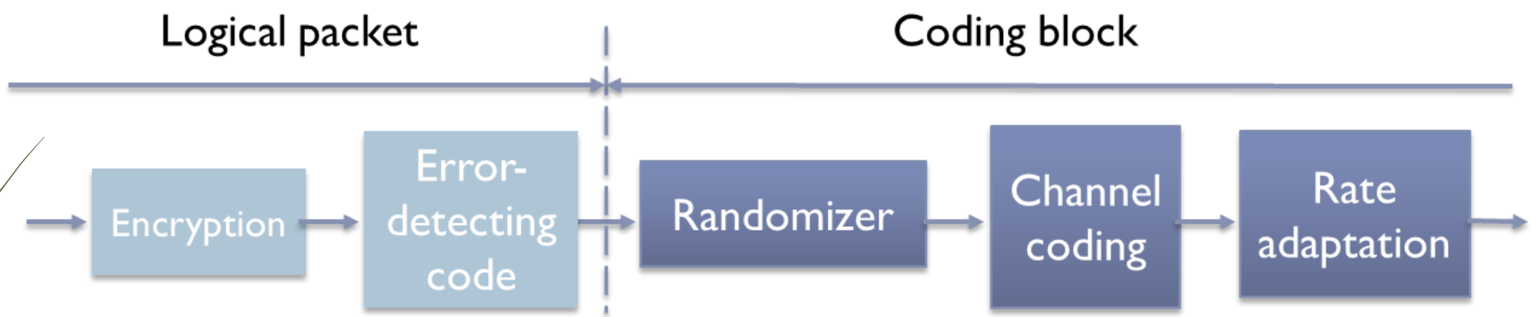
Error detecting code

- Is used by the logical level in order to detect an erroneous logical packet
  - CRC (Cyclic Redundancy Check)

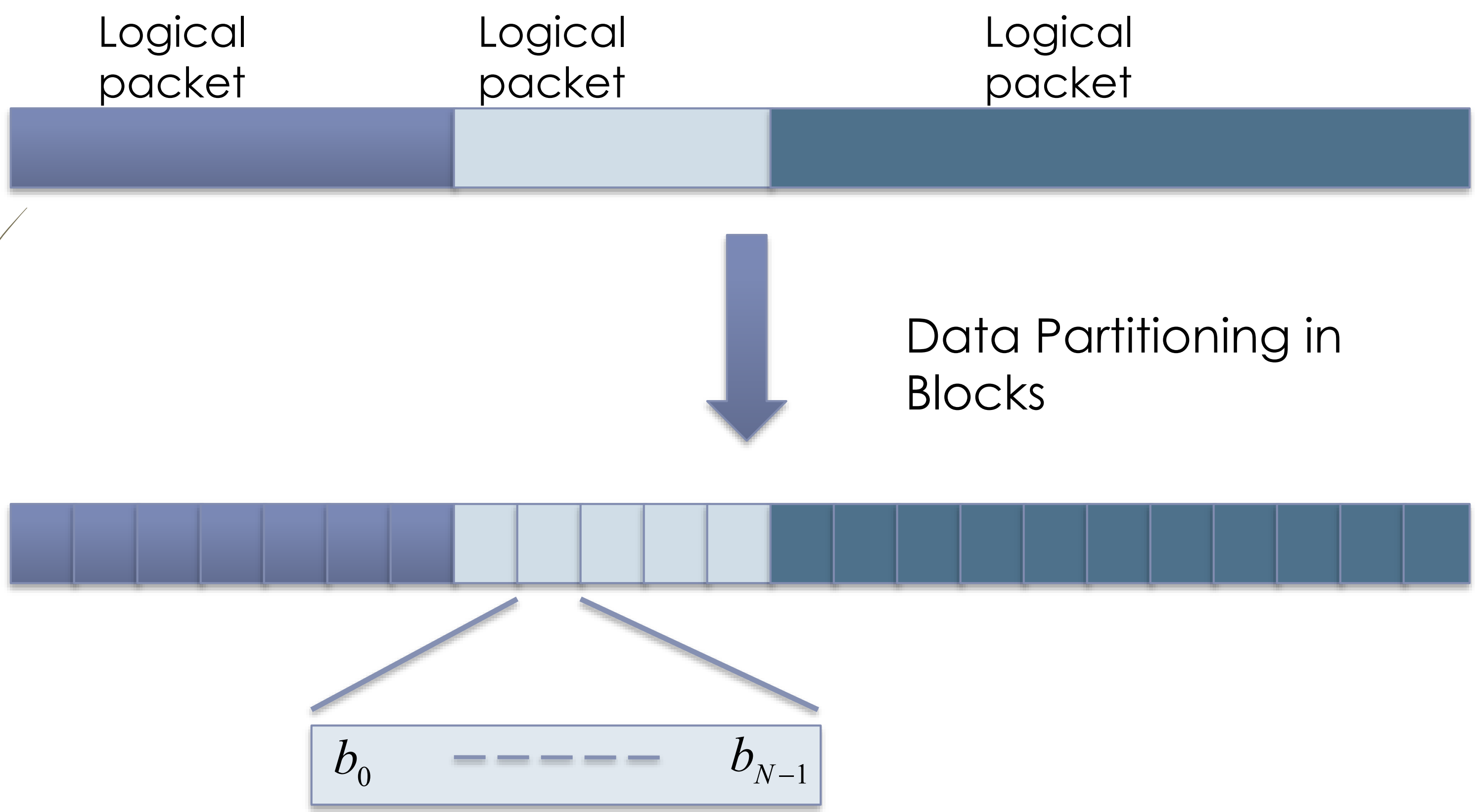


  - Typically used polynomials:
    - CRC-8
    - CRC-16
    - CRC-32
    - CRC-64

Bit-level Processing



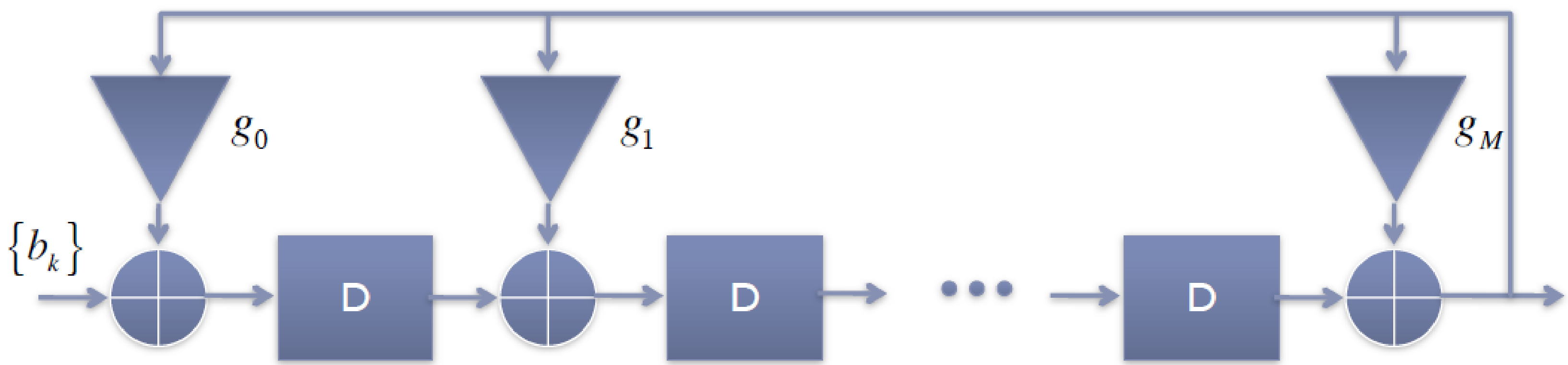
Data Partitioning in Coding Blocks



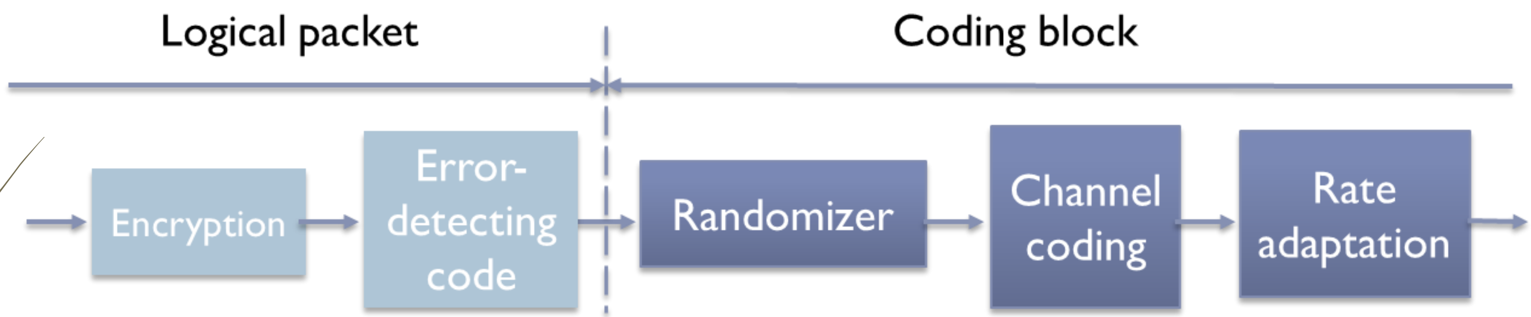
Randomizer

- The randomization operation is made using a **scrambler**
  - Goal: uniformization of 0/1 values
  - Equal energy distribution in time and frequency
- Implementation using a shift register, based on a generator polynomial, g(x)

$$g(x) = g_0 + g_1 x + \ldots + g_M x^M$$



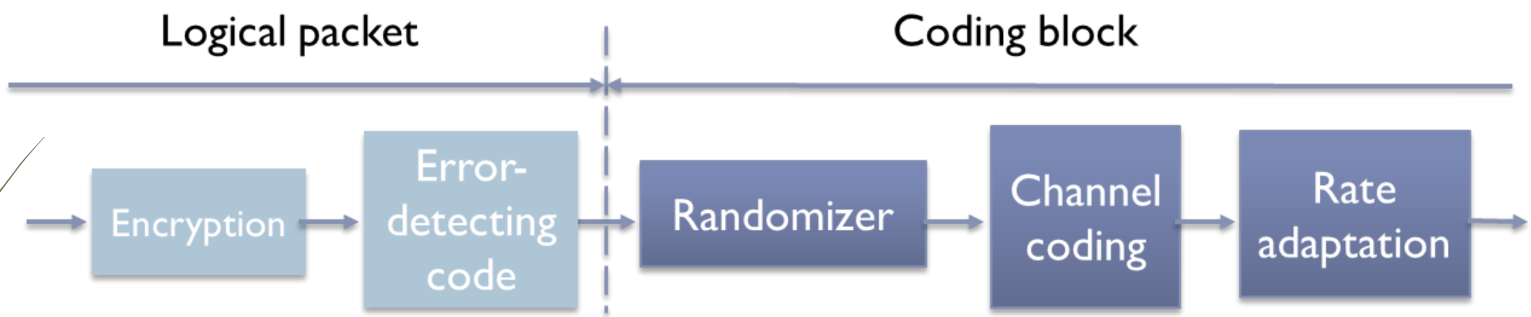
Bit-level Processing



Channel coding

- Addition of redundancy bits
  - Block encoders (Reed Solomon, LDPC)
  - Convolutional encoders (usually preceded by block encoders): CC
  - Turbo encoders (can be standalone): CTC
- Block message:
  - Input size: N
  - Output size: M
  - Code rate: $R_c = \dfrac{N}{M}$

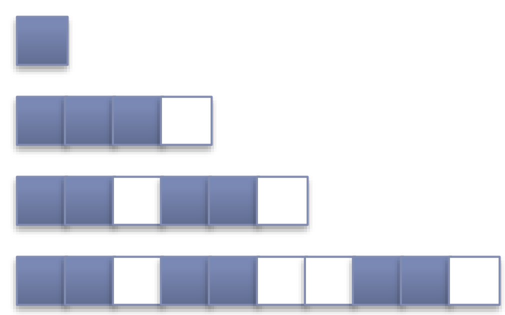
Bit-level Processing



Rate Adaptation

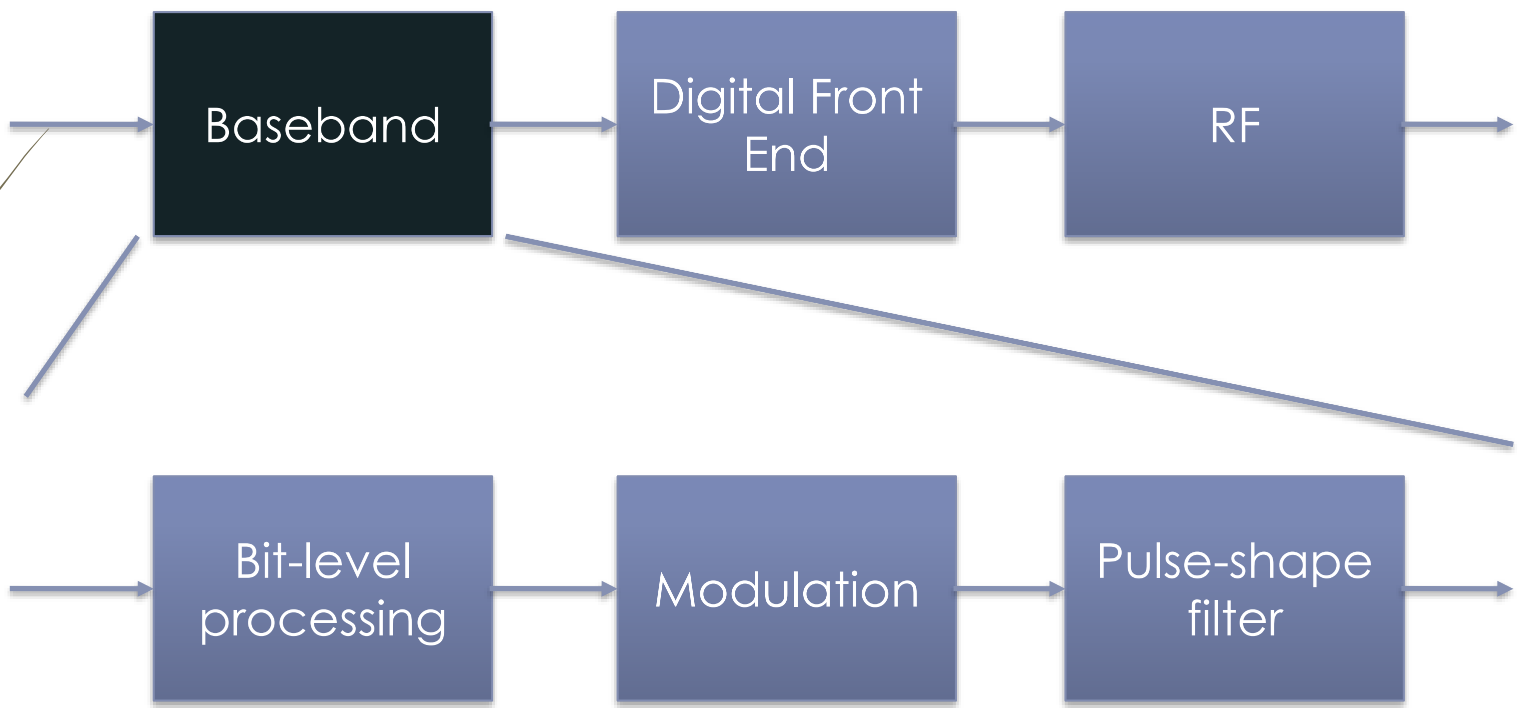
- Encoders offer a fixed rate
  - In case of systems with a variable coding rate, some bits have to be removed
- Puncturing
- Example:
  - CC 802.16d
  - In the resulted code word, the puncturing pattern is repeated

Rată	Pattern
1/2	[1]
2/3	[1 1 1 0]
3/4	[1 1 0 1 1 0]
5/6	[1 1 0 1 1 0 0 1 1 0]



General Aspects

- Baseband processing - Tx

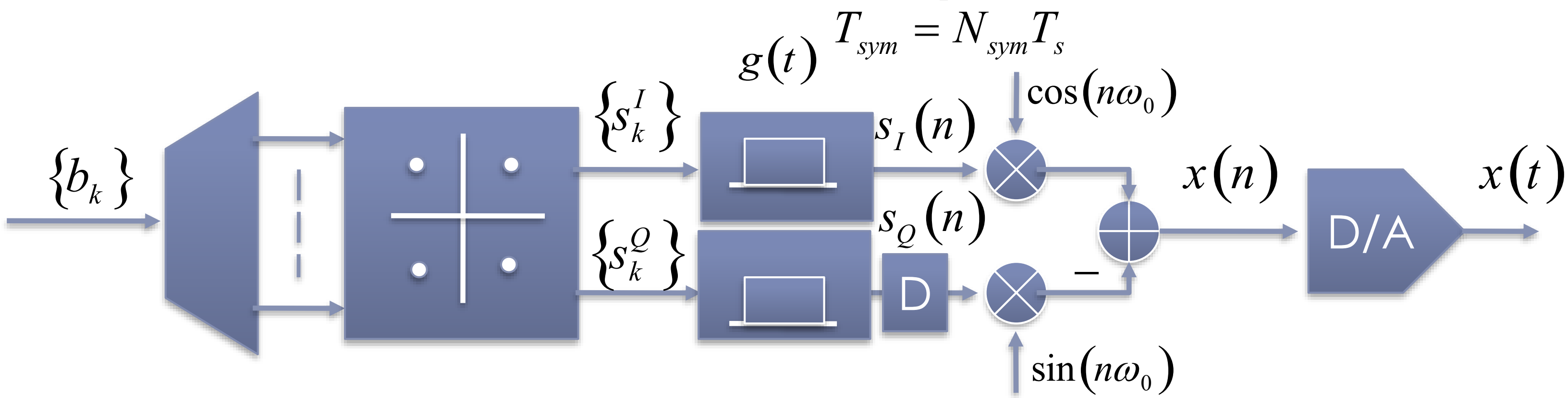


Modulation - Outline

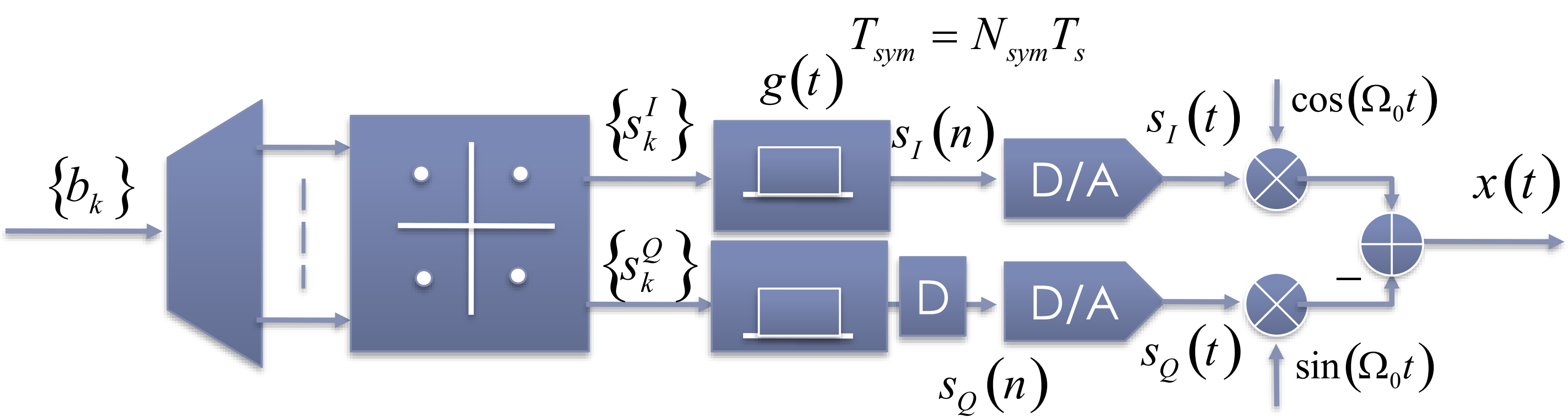
- **Digital amplitude modulation: QAM** (Quadrature Amplitude Modulation)
- **Digital phase modulation: PSK** (Phase-Shift Keying)

- Modulation table (mapper), Modulation rules
- Constellation Diagram of the modulated signal
- Spectral efficiency
- Modulation performance in case of AWGN (Additive White Gaussian Noise)
- Examples of other modulation types

Modulation - QAM



$$s(n) = s_I(n) + js_Q(n) \qquad x(n) = s_I(n)\cos(n\omega_0) - s_Q(n)\sin(n\omega_0) = \mathrm{Re}\left\{s(n)e^{jn\omega_0}\right\}$$

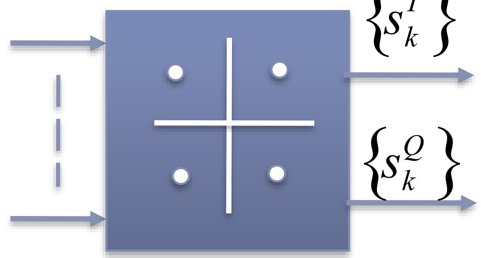


$$s(t) = s_I(t) + js_Q(t) \qquad x(t) = s_I(t)\cos(\Omega_0 t) - s_Q(t)\sin(\Omega_0 t) = \mathrm{Re}\left\{s(t)e^{j\Omega_0 t}\right\}$$

Modulation – QAM (2)

- Mapper (Modulation Table)

$$\{b_i\}, i = \overline{1,P}$$



$$\{s_k^I\} \qquad \{s_k^Q\}$$

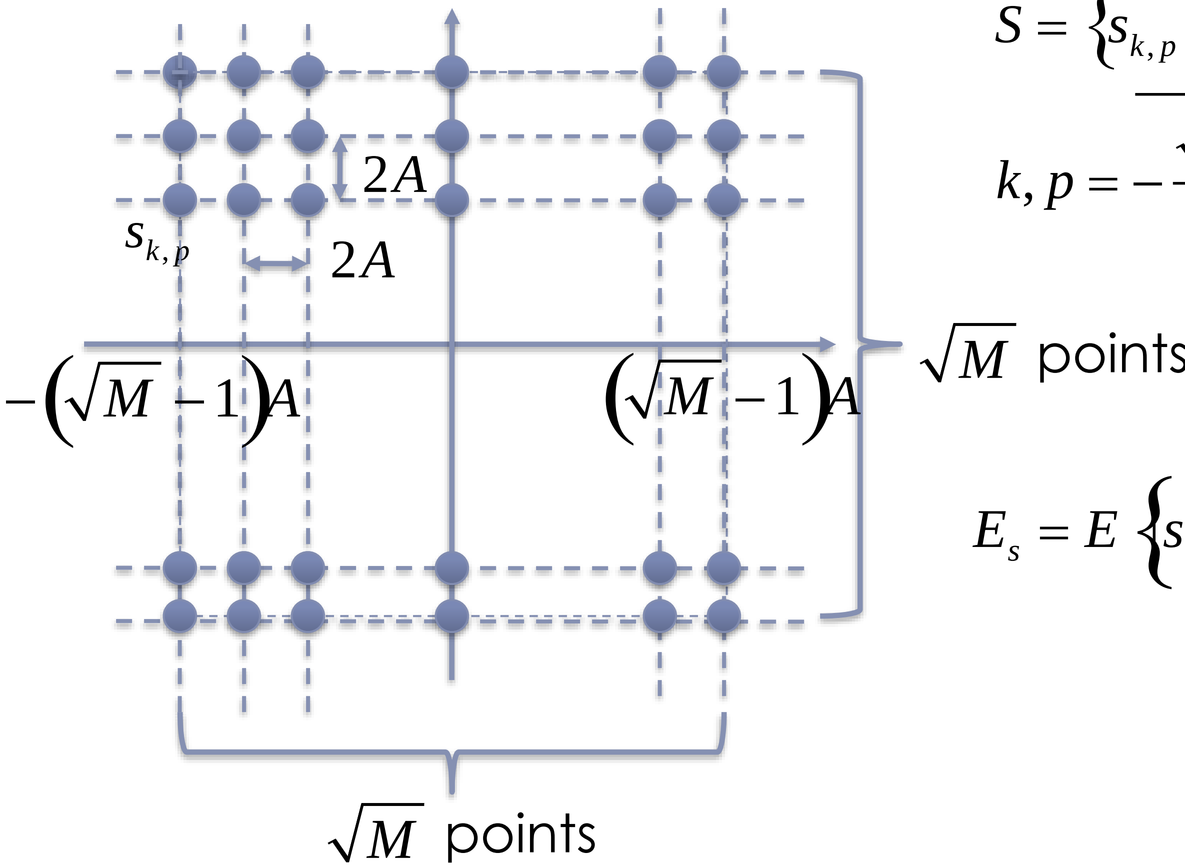
$$|s_k|^2 = (s_k^I)^2 + (s_k^Q)^2$$

- Modulation Rules

$$E\left\{|s_k|^2\right\} = \frac{E_s}{T_{sym}} \qquad E\{s_k\} = 0$$

$$s_k \in S \qquad card\{S\} = 2^P = M$$

Modulation – QAM (3)



$$S = \left\{ s_{k,p} = (2k+1) + j(2p+1) \right\}$$

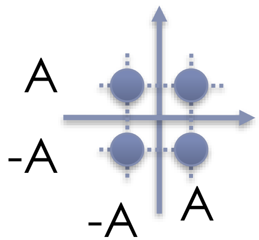
$$k,p = -\frac{\sqrt{M}}{2}, \left(\frac{\sqrt{M}}{2} - 1 \right)$$

$$E_s = E\left\{ \left| s_{k,p} \right|^2 \right\} T_{sym} = \frac{2A^2}{3}(M-1)T_{sym}$$

$$E_b = \frac{E_s}{\log_2 M}$$

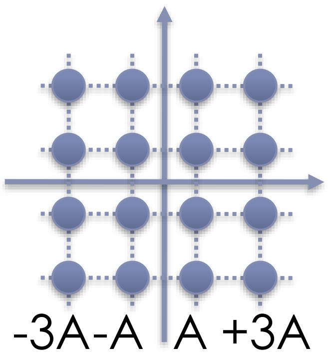
Modulation – QAM (4)

4-QAM



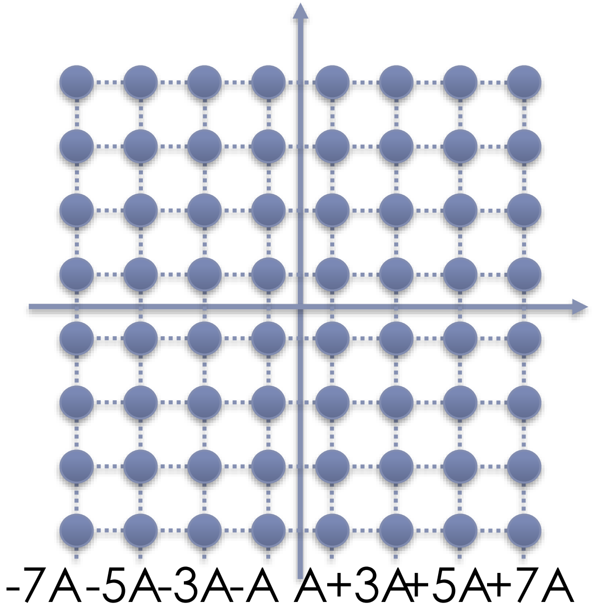
$$E_s = 2A^2 T_{sym}$$

16-QAM



$$E_s = 10A^2 T_{sym}$$

64-QAM



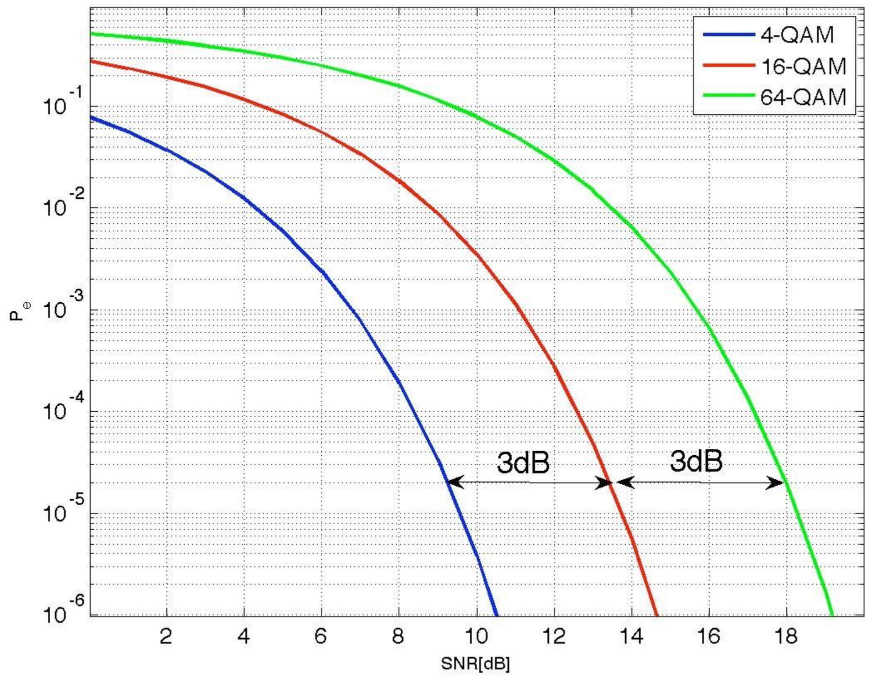
$$E_s = 42A^2 T_{sym}$$

- The signals have the same spectrum as the PSK signals with the same symbol period

QAM Modulation performance in AWGN

- Symbol error probability:

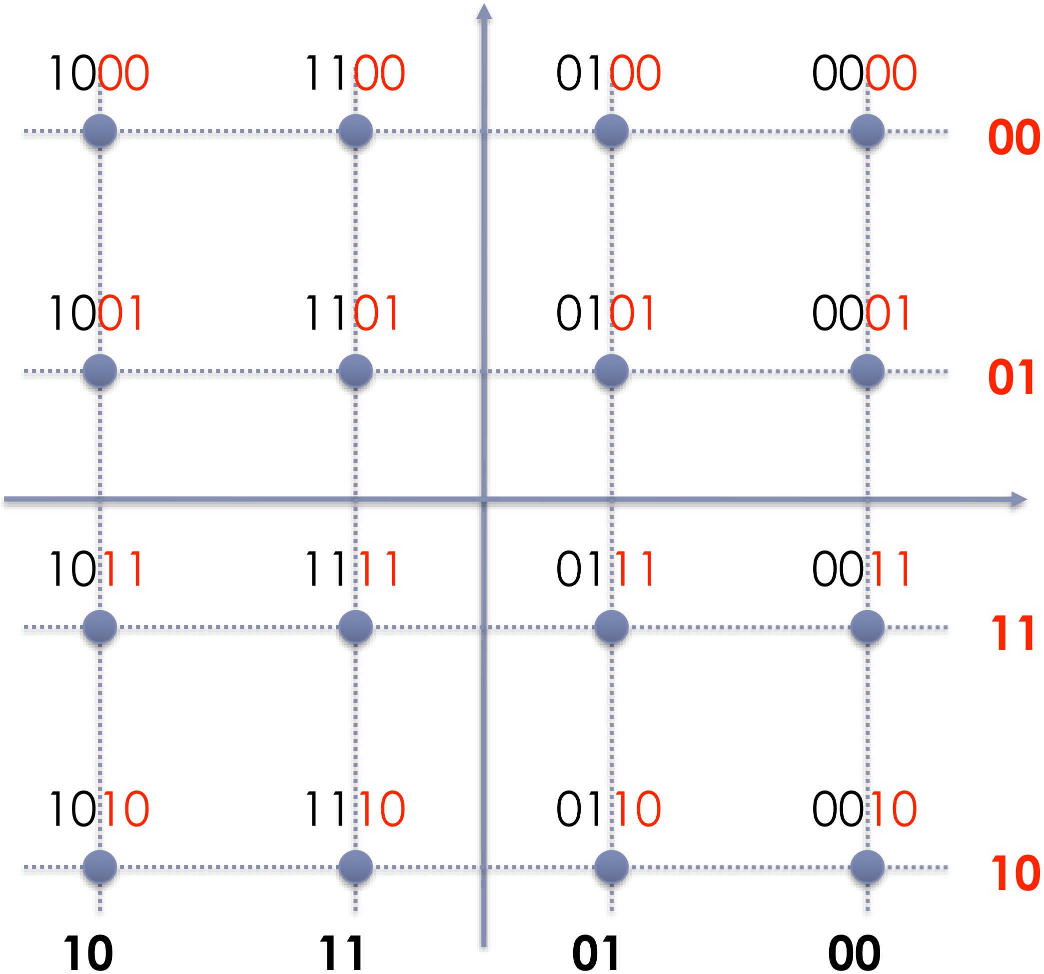
$$P_{e,s} = 1 - \left[1 - \frac{2\left(\sqrt{M} - 1\right)}{\sqrt{M}} Q\left(\sqrt{\frac{3\log_2 M}{M-1} \frac{E_s}{N_0}} \right) \right]$$



- Bit error probability:

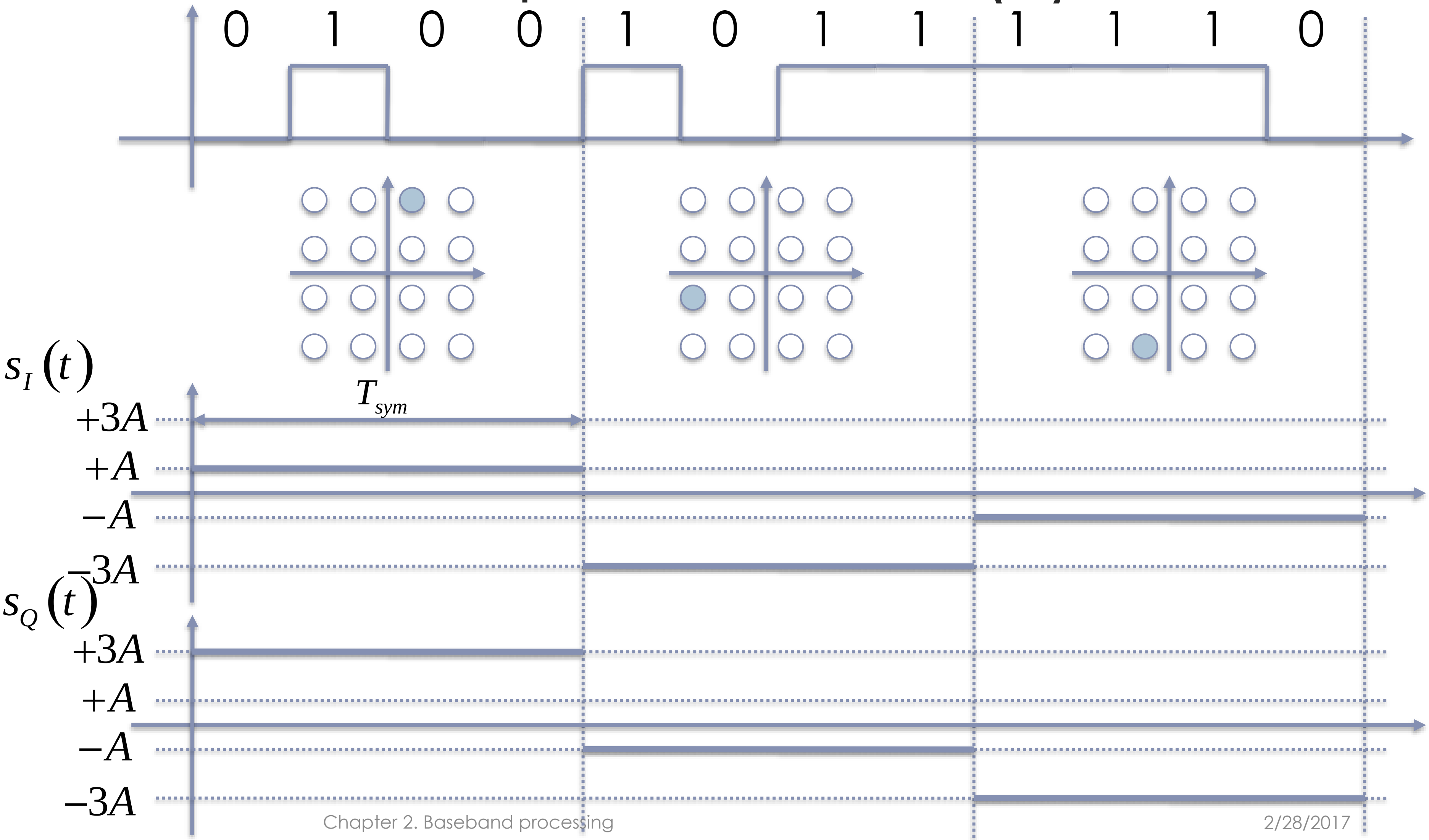
$$P_{e,s} = \frac{P_{e,b}}{\log_2 M}$$

Example: 16-QAM

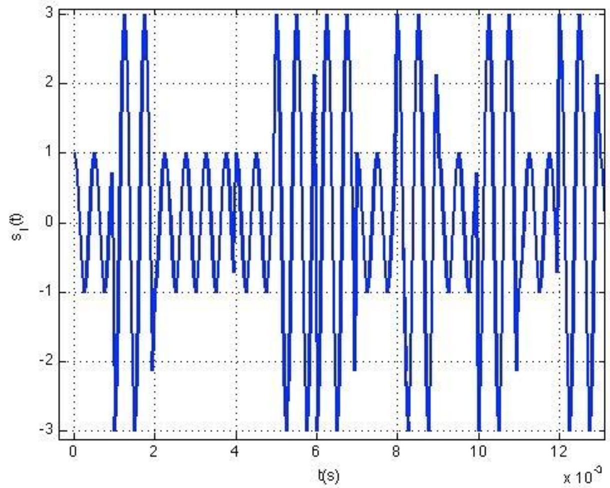


Example of Gray coding

Example: 16-QAM (2)

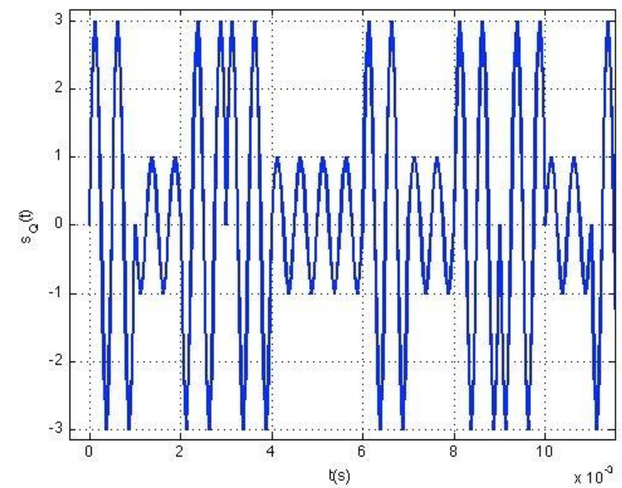


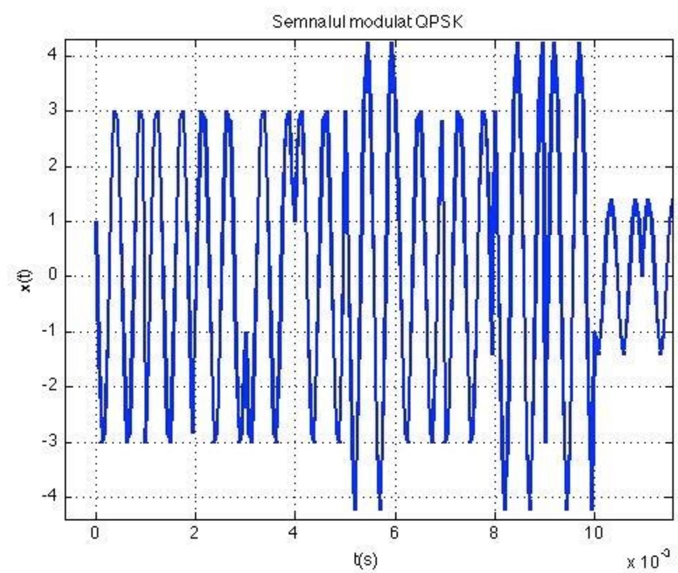
Example: 16-QAM (3)



$s_I(t)$

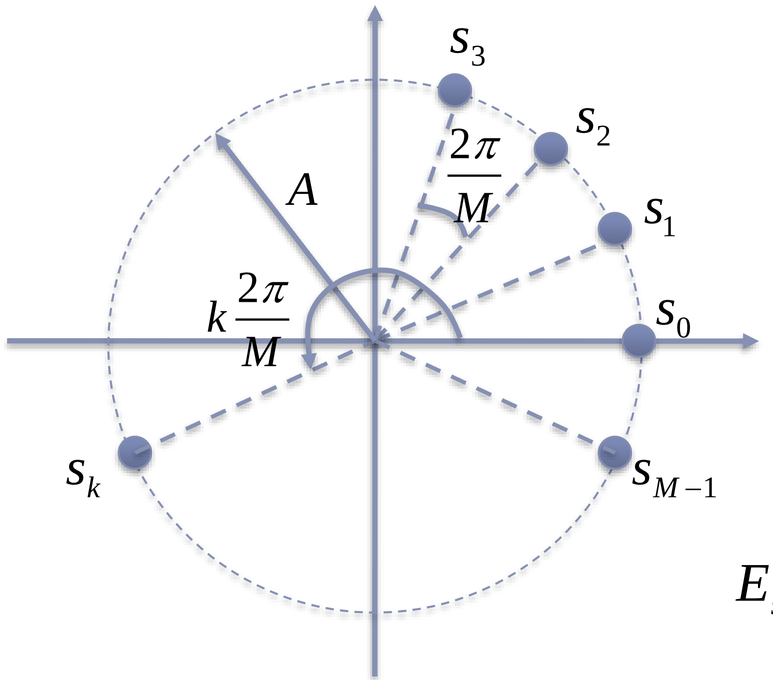
$s_Q(t)$





$x(t)$

Modulation - PSK



$$S = \left\{Ae^{j\varphi_k}\right\}, k = \overline{0, M-1}$$

$$\varphi_k = k\frac{2\pi}{M}$$

$$s_k = Ae^{jk\frac{2\pi}{M}}, k = \overline{0, M-1}$$

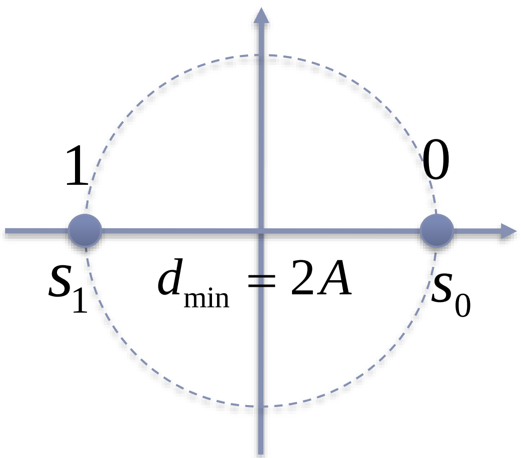
$$E_s = \left|s_k\right|^2 T_{sym} = A^2 T_{sym} \qquad E_b = \frac{E_s}{\log_2 M}$$

$$\varphi(n) = \arg\{s(n)\} = \sum_{p=-\infty}^{\infty} \varphi_p g\left(n - pN_{sym}\right)$$

$$x(n) = A\cos\left(n\omega_0 + \sum_{p=-\infty}^{\infty} k_p \frac{2\pi}{M} g\left(n - pN_{sym}\right)\right)$$

Modulation – PSK (2)

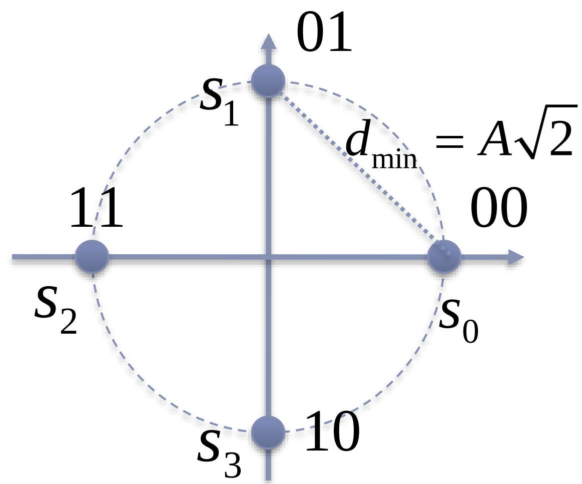
BPSK



$$S = \{\pm A\}$$

$$\varphi_k \in \{0, \pi\}$$

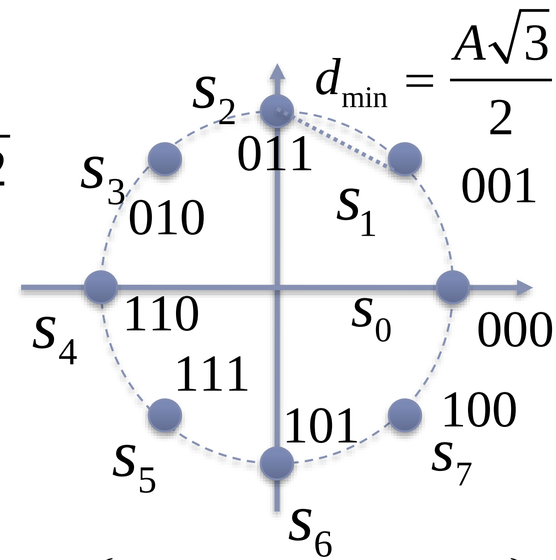
4-PSK



$$S = \{\pm A, \pm jA\}$$

$$\varphi_k \in \left\{0, \frac{\pi}{2}, \pi, -\frac{\pi}{2}\right\}$$

8-PSK

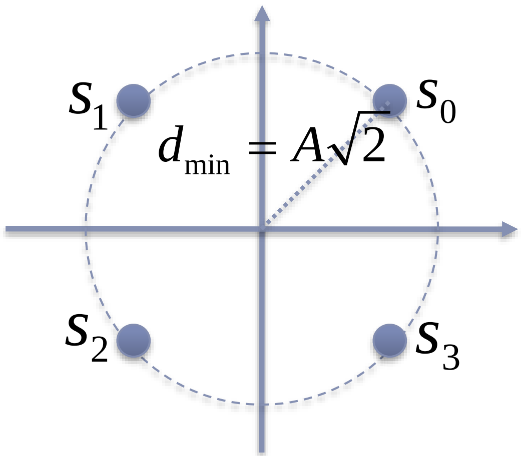


$$S = \left\{\pm A, \pm jA, \pm \frac{A}{\sqrt{2}} \pm \frac{jA}{\sqrt{2}}\right\}$$

$$\varphi_k \in \left\{0, \pm\frac{\pi}{2}, \pi, \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}\right\}$$

Modulation – PSK (3)

QPSK



$$S = \left\{ \pm \frac{A}{\sqrt{2}} \pm \frac{jA}{\sqrt{2}} \right\}$$

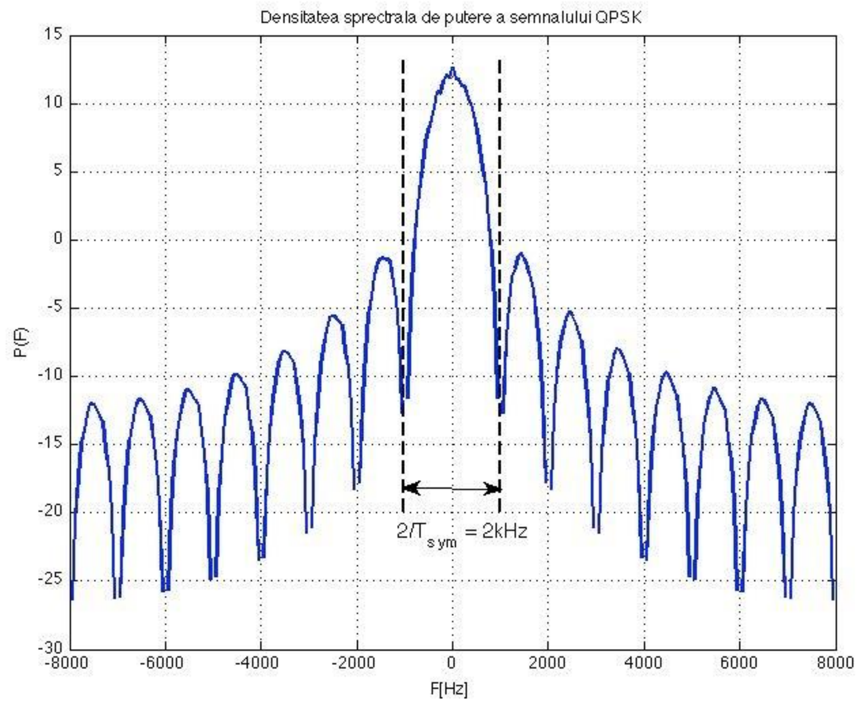
$$\varphi_k \in \left\{ \pm \frac{\pi}{4}, \pm \frac{3\pi}{4} \right\}$$

Variation of 4-PSK, by means of a rotation with $\pi/4$

Modulation – PSK (4)

- Spectral density of the analytical signal:

$$s(t) = s_I(t) + js_Q(t)$$

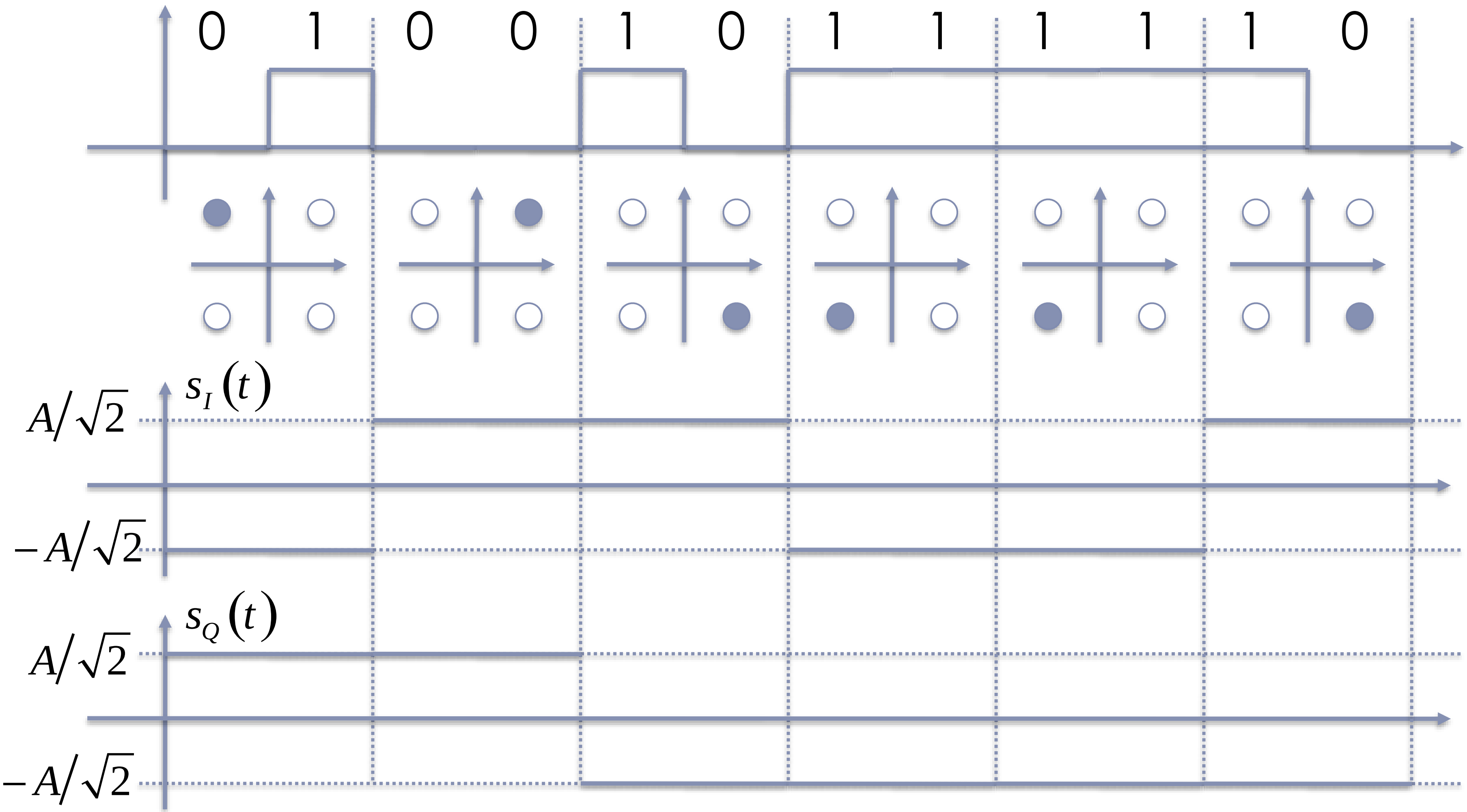


$$B = \frac{2}{T_s} \qquad B_{90\%} = \frac{1.7}{T_s} \qquad B_{99\%} = \frac{20}{T_s}$$

Spectral efficiency:

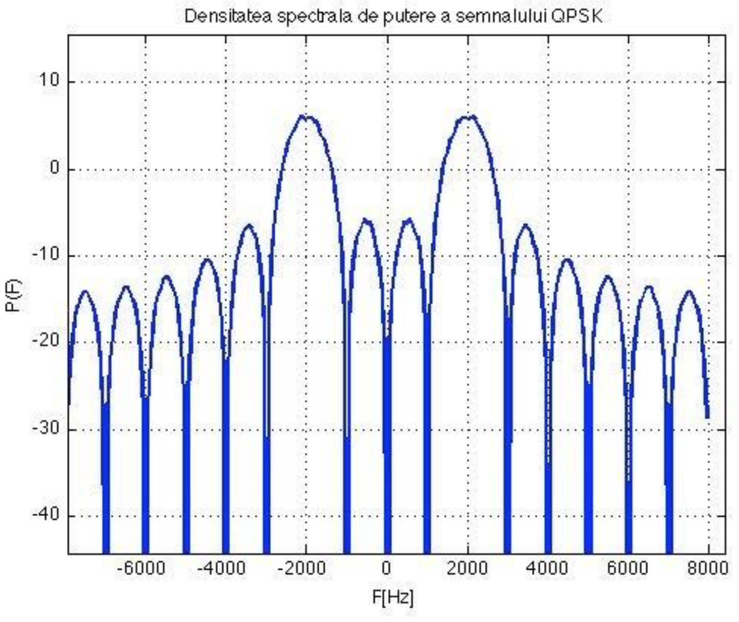
$$\eta = \frac{R_b}{B} = \frac{\frac{1}{T_b}}{\frac{2}{T_{sym}}} = \frac{\frac{1}{T_b}}{\frac{2}{(\log_2 M)T_b}} = \frac{\log_2 M}{2}$$

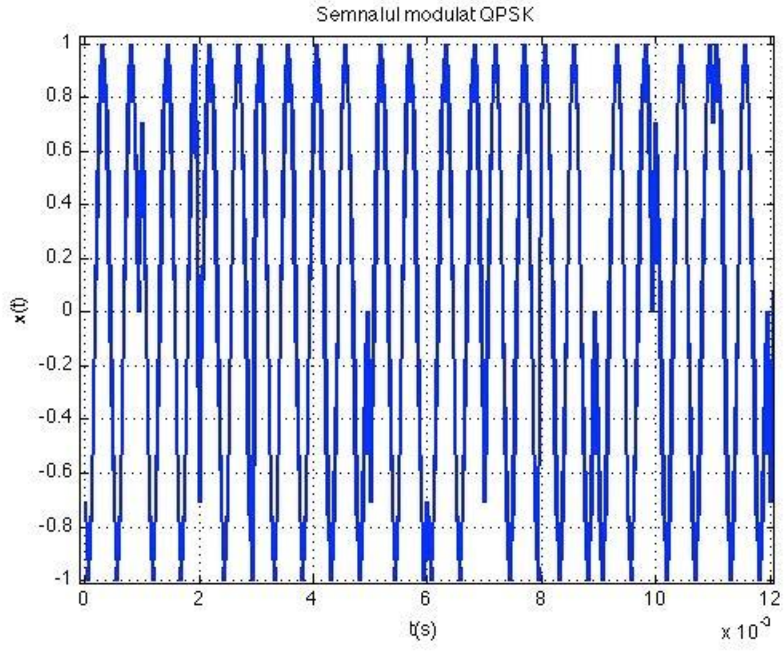
Example: QPSK Communication



Example: QPSK Communication (2)

- $F_0 = 4\text{kHz} = F_s \ / \ 4$





Example: QPSK Communication (3)

- The signal is generated in discrete time
- What sampling frequency can be used ?

$$F_s \geq 2F_M$$ **The Nyquist condition**

- The QPSK signal has an unlimited bandwidth
  - F_M has to be approximated
  - The equivalent bandwidth has to be considered
  - The sampling frequency cannot be chosen exactly the Nyquist frequency because of the aliasing phenomenon
  - We choose $F_s >> 2F_M$
- T_{sym} = 1ms, T_b = 500ms

$$B \approx \frac{2}{T_{sym}} = 2kHz \qquad F_M \approx 1kHz$$

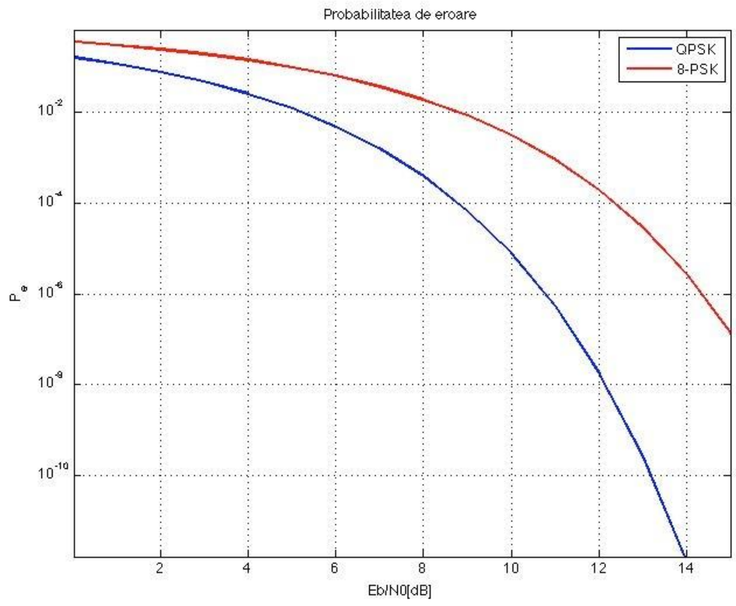
$$F_s = 16kHz$$

PSK Modulation performance in AWGN

$$P_{e,M-PSK} = 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}}\left(\sin\frac{\pi}{M}\right)\right)$$

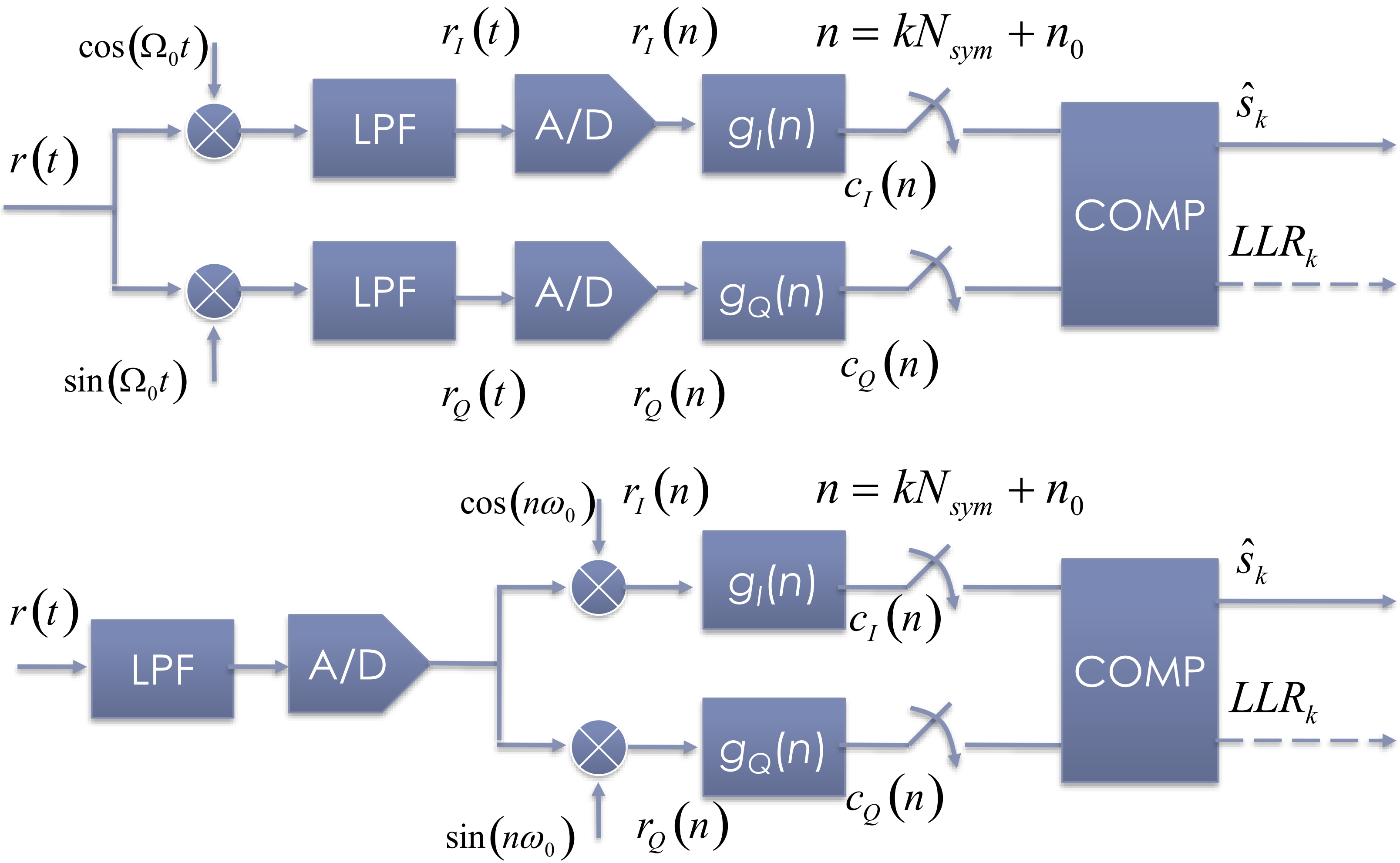
$$P_{e,BPSK} = P_{e,QPSK} = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_{e,8-PSK} = 2Q\left(0.937\sqrt{\frac{E_b}{N_0}}\right)$$



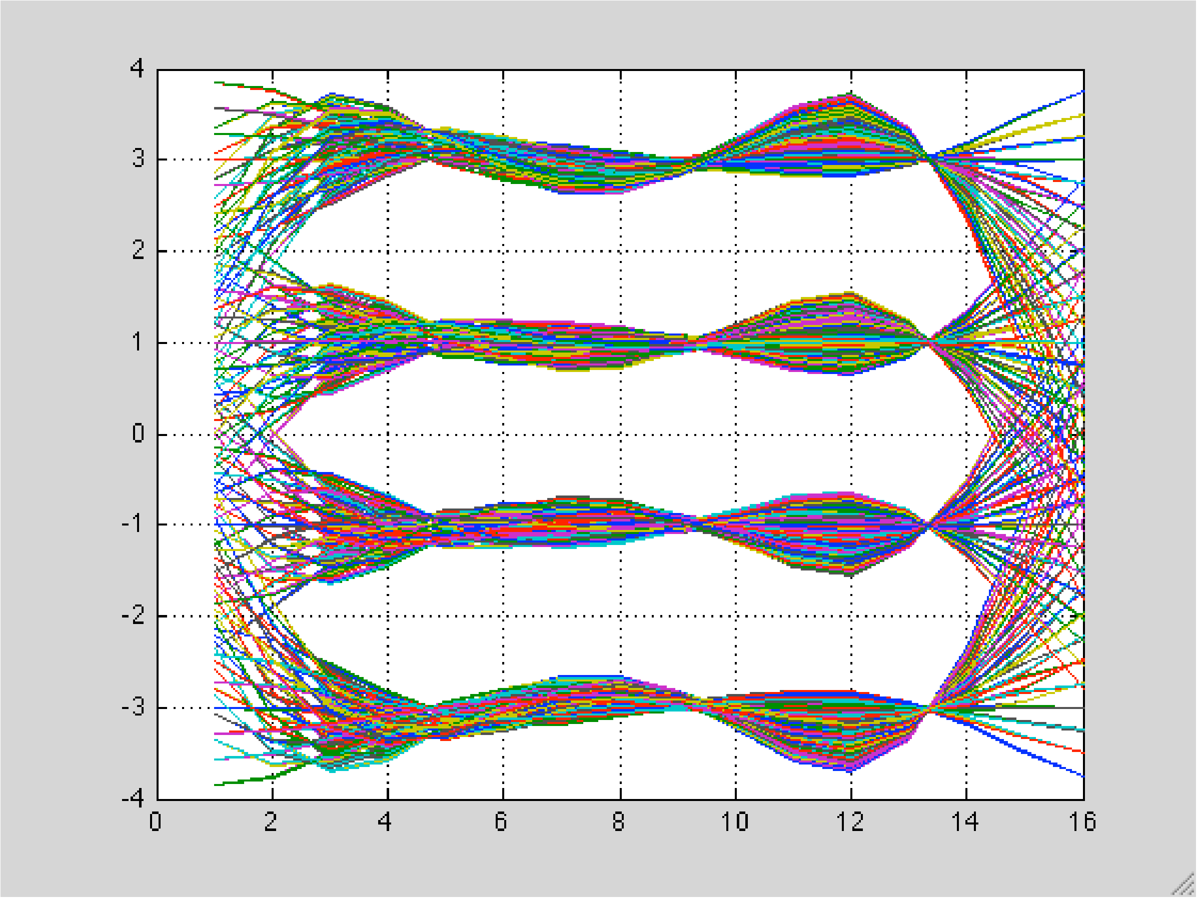
Coherent receiver

Demodulation – QAM signals



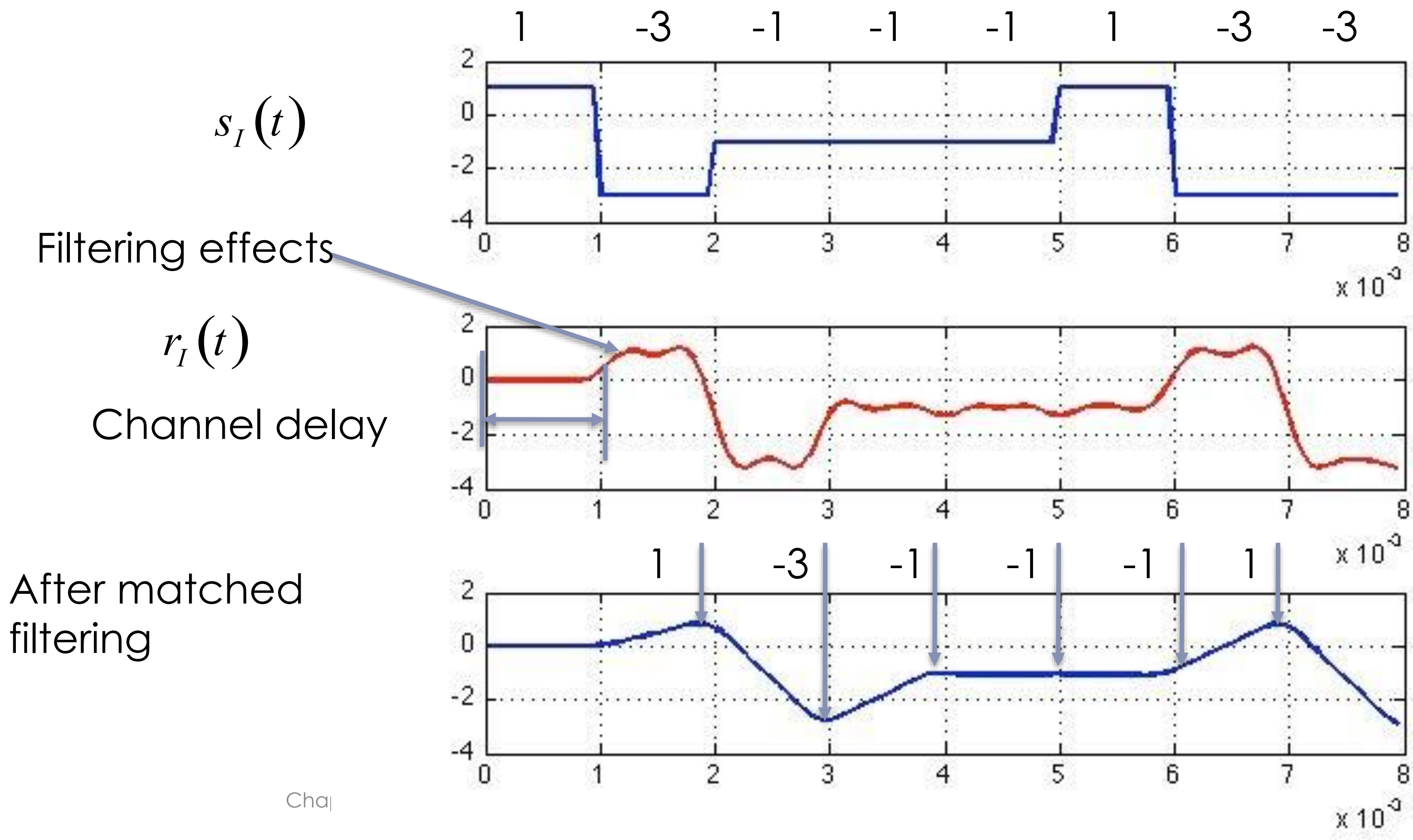
Example: 16-QAM

- Eye diagram:



Example: 16-QAM (2)

- AWGN: detection of the real part



Hard decisions

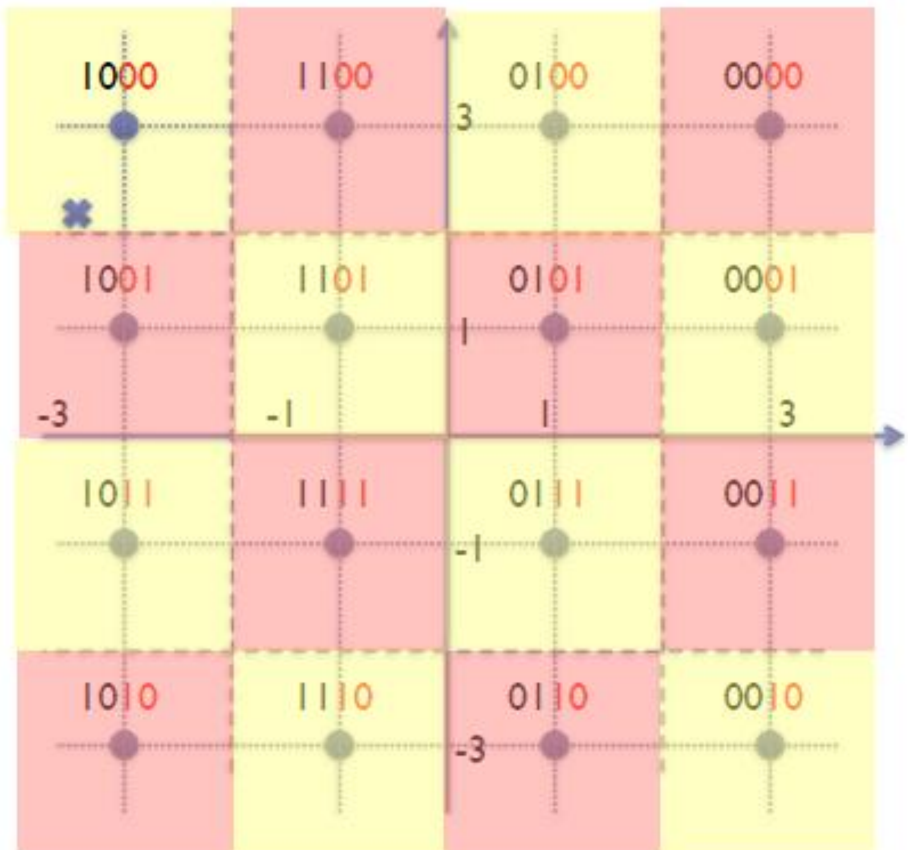
- The symbol from the constellation that is closest to the received one is determined
- Each symbol from the constellation has a corresponding decision region
  - Any point from the decision region of the $\boldsymbol{s}_k$ symbol has the $\boldsymbol{s}_k$ symbol the closest from all the symbols in the constellation

$$\hat{s}_k = \arg\min_{\substack{k \\ s_k \in S}} \sum |r - s_k|^2$$

- For QAM modulations, the real and imaginary parts can be separated

$$\hat{s}_k^I = \arg\min_{\substack{k \\ s_k \in S}} \sum \left|\mathrm{Re}\{r\} - s_k^I\right|^2 \qquad \hat{s}_k^Q = \arg\min_{\substack{k \\ s_k \in S}} \sum \left|\mathrm{Im}\{r\} - s_k^Q\right|^2$$

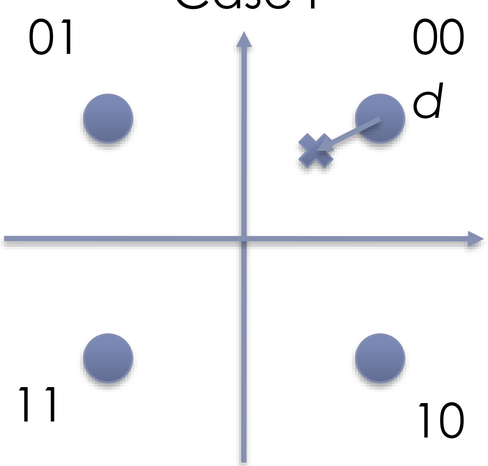
Hard decisions (2)



LLR (Log-Likelihood Radio)

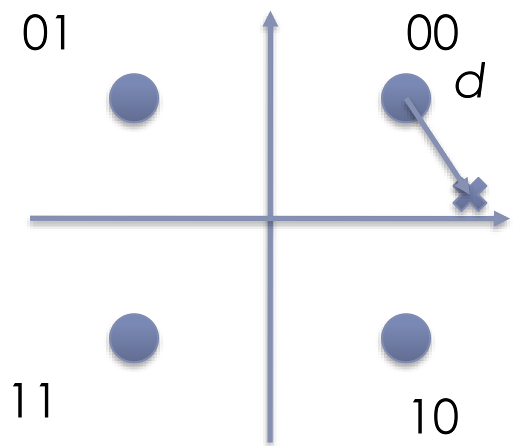
- Likelihood ratios in the logarithmic domain;
- Example: 2 cases for a QPSK received signal
- The received symbol is located in two possible positions

Case I



$$\hat{s}_k = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \qquad \hat{b}_{k,0} = 0 \qquad \hat{b}_{k,1} = 0$$

Case II



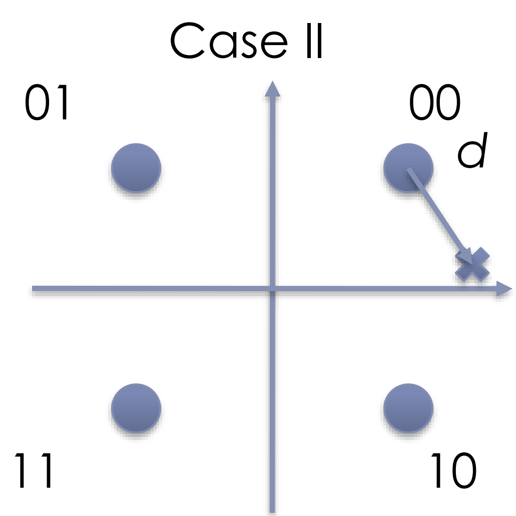
$$\hat{s}_k = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \qquad \hat{b}_{k,0} = 0 \qquad \hat{b}_{k,1} = 0$$

LLR (Log-Likelihood Radio) (2)

- In both cases, the same symbol is detected;
  - However, in the 2nd case the noise is higher, so the probability to obtain a correct decision is lower
  - Another indicator should be given, related to the SNR or to the decision likelihood
- Decisions:
  - **Hard:** the detected symbol is reported
  - **Soft:** the trust in the obtained vales is reported

LLR (Log-Likelihood Radio) (3)

- Bit b_0 is at the decision border between 0 and 1
- Bit b_1 is highly probable to be 0
- The likelihoods of the two bits should be differentiated.



LLR (Log-Likelihood Radio) (4)

- The likelihood ratio

$$LR = \frac{P\left(b_k = 1 \middle| r(t)\right)}{P\left(b_k = 0 \middle| r(t)\right)}$$

- The log-likelihood ratio

$$LLR = \ln\left[\frac{P\left(b_k = 1 \middle| r(t)\right)}{P\left(b_k = 0 \middle| r(t)\right)}\right]$$

- In case of AWGN, the LLRs are calculated with the following algorithm.

LLR (Log-Likelihood Radio) (5)

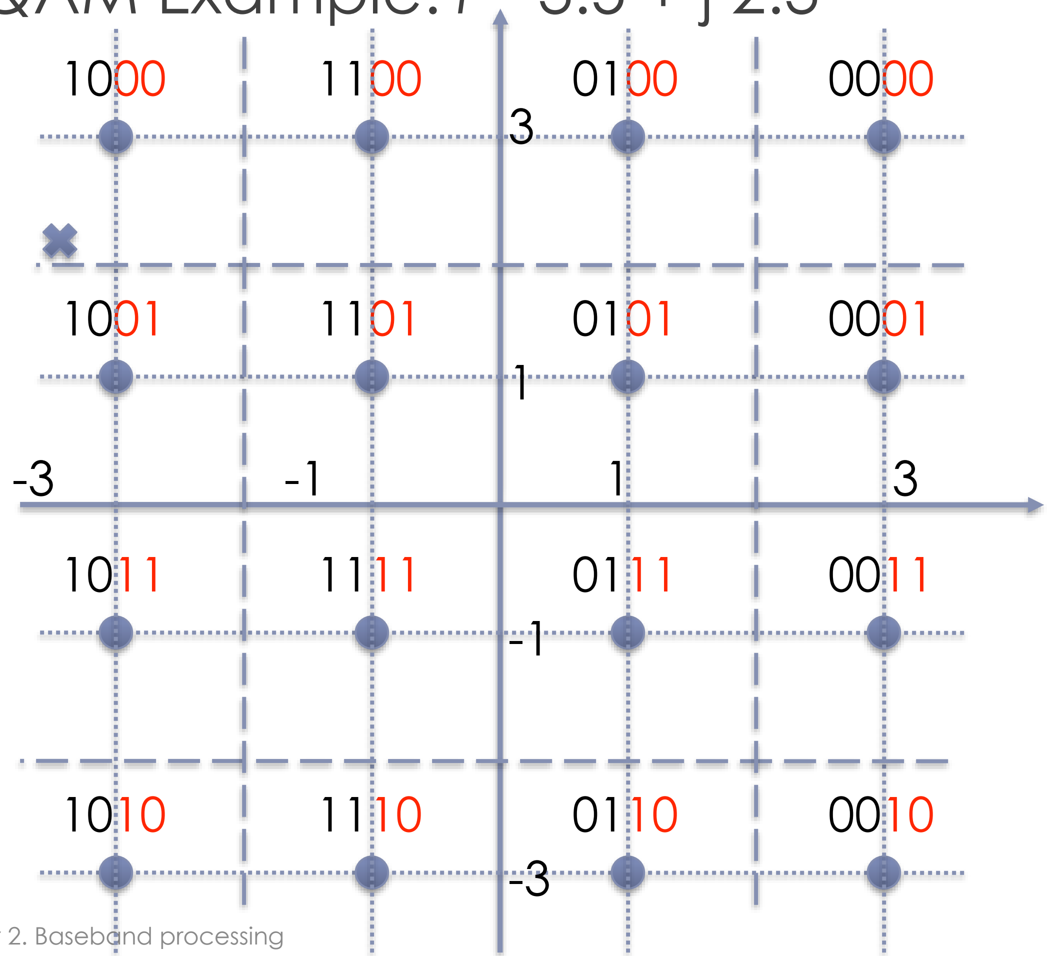
- For bit *k* of the received symbol *r(n)*
- All the symbols from the *S* constellation, s_q which have bit k on 1 $\Rightarrow S_{k,1}$
- We are looking for all the symbols s_q which have the k bit on 0 $\Rightarrow S_{k,0}$

$$LLR\left(b_{I,k}(n)\right) = \ln \frac{\sum_{s_q \in S_{k,1}} p\left(r(n) \middle| s(n) = s_q\right)}{\sum_{s_q \in S_{k,0}} p\left(r(n) \middle| s(n) = s_q\right)} \approx \ln \frac{\max_{s_q \in S_{k,1}} p\left(r(n) \middle| s(n) = s_q\right)}{\max_{s_q \in S_{k,0}} p\left(r(n) \middle| s(n) = s_q\right)}$$

$$LLR\left(b_{I,k}(n)\right) = \frac{1}{4}\left(\min_{s_q \in S_{k,0}} \left|r(n) - s_q\right|^2 - \min_{s_q \in S_{k,1}} \left|r(n) - s_q\right|^2\right)$$

LLR (Log-Likelihood Radio) (6)

- 16-QAM Example: r=-3.5 + j 2.3



LLR (Log-Likelihood Radio) (7)

- Linear approximation
  - 16-QAM Example:

$$LLR_3 = \begin{cases} -r_I, |r_I| \le 2 \\ 2(-r_I - 1), |r_I| > 2 \\ 2(-r_I + 1), |r_I| < -2 \end{cases} \quad LLR_1 = \begin{cases} -r_Q, |r_Q| \le 2 \\ 2(-r_Q - 1), |r_Q| > 2 \\ 2(-r_Q + 1), |r_Q| < -2 \end{cases} \quad \begin{aligned} LLR_2 &= 2 - |r_I| \\ LLR_0 &= 2 - |r_Q| \end{aligned}$$

  - If *r* = -3.5 + j2.3
    - Hard decision:

$$\hat{s} = -3 + j3 \quad b_3 = 1, b_2 = 0, b_1 = 0, b_0 = 0$$

    - Soft decision:

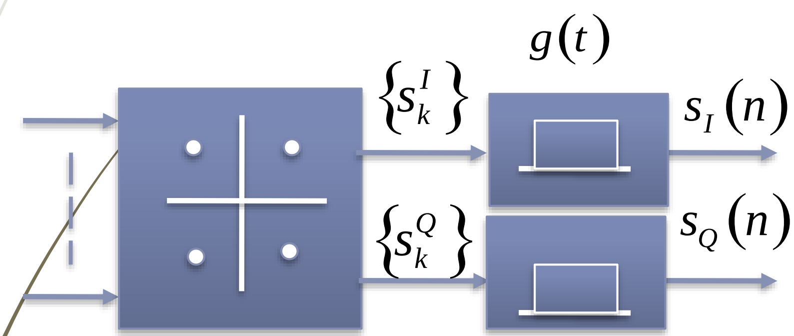
$$LLR_3 = 9, LLR_2 = -1.5, LLR_1 = -6.6, LLR_0 = -0.3.$$

Pulse-shape filtering

- Purpose:
  - Reducing the necessary transmitted signal bandwidth
    - Increase in spectral efficiency
  - Reducing the inter-symbol interference
- Methods:
  - Using an analog or digital pulse of a certain shape
  - Introducing a pulse-shaping filter on the transmit signal path

Pulse-shape filtering

- In the initial transmission diagram:



$$s_I(n) = \sum_{k=-\infty}^{\infty} s_k^I g(n - kN_{sym})$$

$$s_Q(n) = \sum_{k=-\infty}^{\infty} s_k^Q g(n - kN_{sym})$$

- The pulse-shaping filter ensures an oversampling of N_{sym}

$$F_s = N_{sym} F_{sym}$$

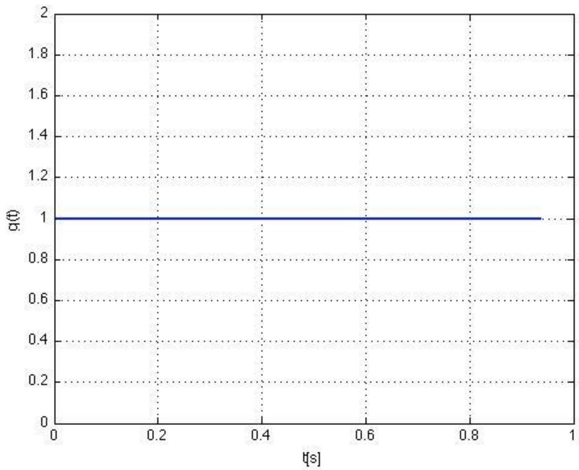
Pulse-shape filtering

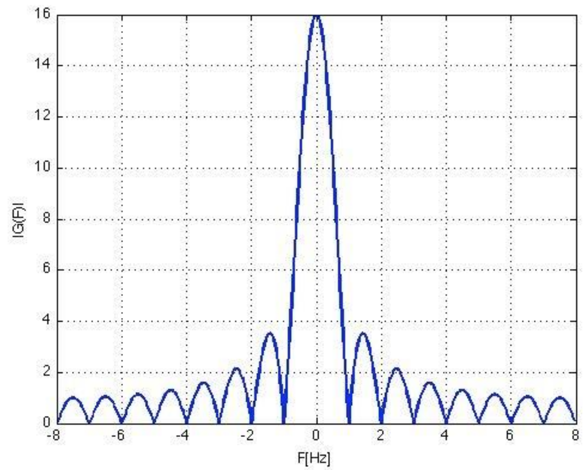
- Examples of pulse-shaping filters:
  - Sinc filter (rectangular pulse)
  - Rectangular filter (sinc pulse)
  - Raised cosine filter (RC)
  - Root raised cosine filter (RRC)
  - Triangular pulse
  - Gaussian pulse

Rectangular pulse

$$g(t) = u(t) - u(t - T_{sym})$$

$$|G(F)| = AT_s \mathrm{sinc}(\pi F T_{sym})$$





- Infinite bandwidth
- In practice, the transmit filters limit the bandwidth of the signal
  - The pulses lose their rectangular shape
- The band is occupied in an inefficient way

Sinc pulse

$$g(t) = \mathrm{sinc}\left(\frac{\pi\left(t - \frac{T_{sym}}{2}\right)}{T_{sym}}\right) \qquad |G(F)| = \begin{cases} 1, F \in \left[-\frac{F_{sym}}{2}, \frac{F_{sym}}{2}\right] \\ 0, F \notin \left[-\frac{F_{sym}}{2}, \frac{F_{sym}}{2}\right] \end{cases}$$

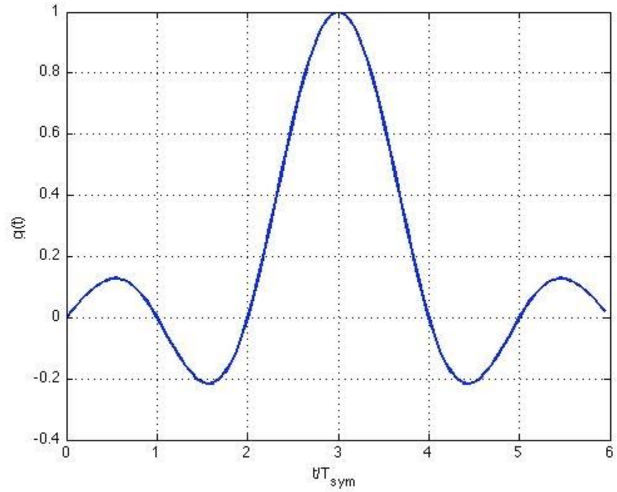
- Physically unattainable (non-causal, infinite support)
- The truncation of *g(t)* and the delay of the pulse are necessary

$$g(t) = \mathrm{sinc}\left(\frac{\pi\left(t - \frac{T_0}{2}\right)}{T_{sym}}\right)\left(u(t) - u(T_0)\right)$$

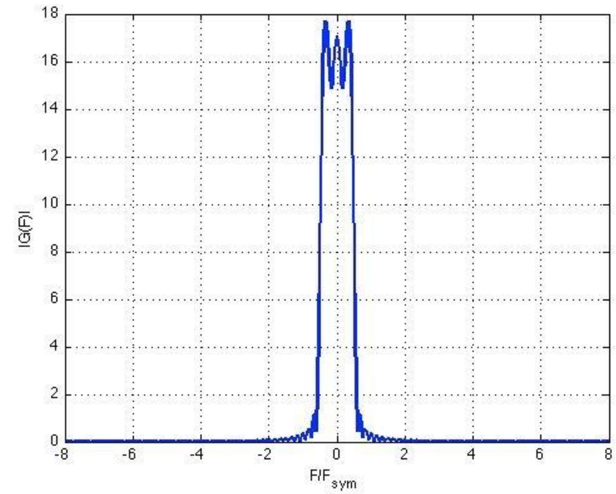
- Respects the Nyquist criterion for avoiding ISI (Inter Symbol Interference)

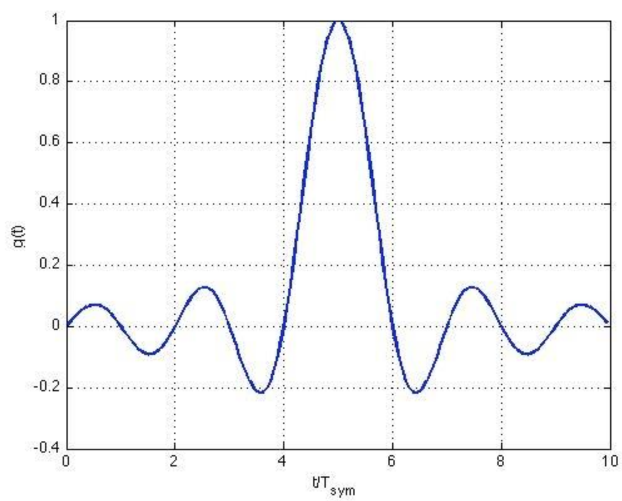
$$g(kT_{sym}) = \delta(k)$$

Sinc pulse (2)

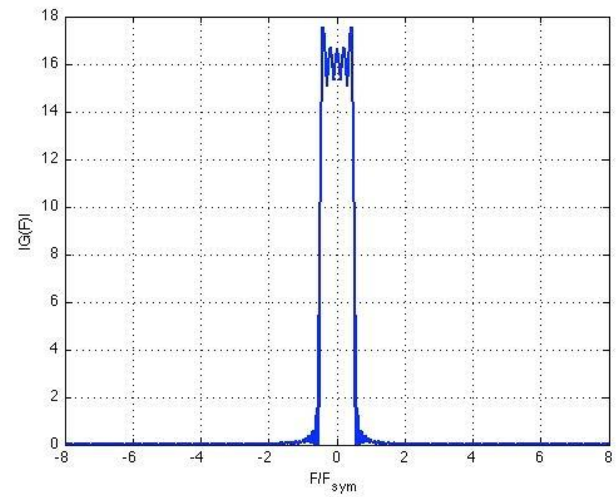


$$T_0 = 6T_{sym}$$





$$T_0 = 10T_{sym}$$



Raised cosine filter (RC)

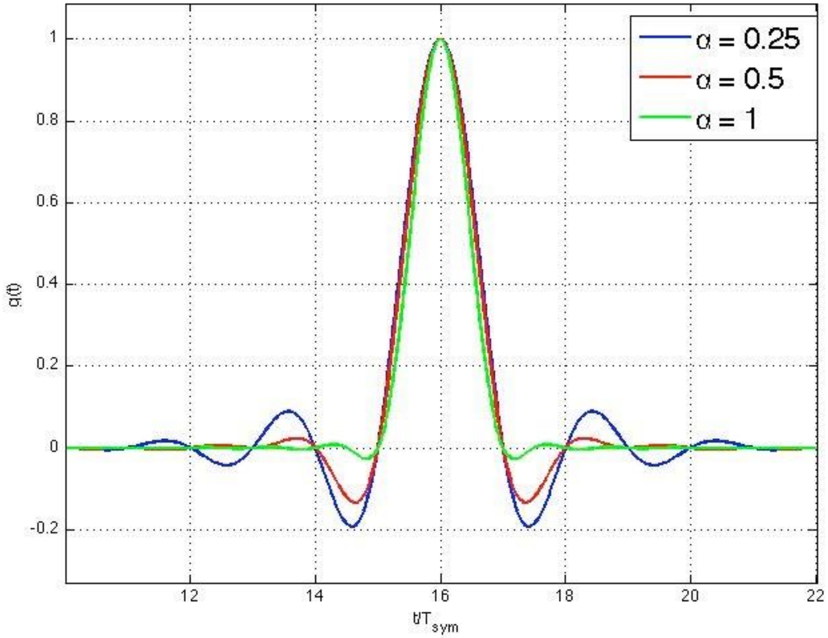
$$g\left(t+\frac{T_{sym}}{2}\right)=\frac{\cos\left(\frac{\alpha\pi t}{T_{sym}}\right)}{1-\left(\frac{2\alpha t}{T_{sym}}\right)^{2}}\operatorname{sinc}\left(\frac{\pi t}{T_s}\right)$$

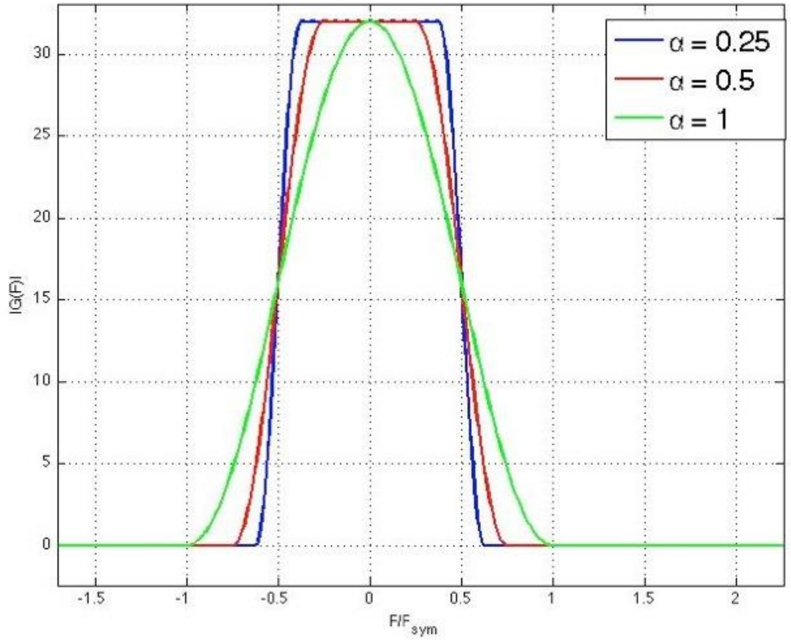
$$G(F)=\begin{cases} T_s, |F|\le\frac{1-\alpha}{2T_{sym}} \\ T_s\frac{1-\sin\left(\frac{\pi}{2\alpha}\left(2FT_{sym}-1\right)\right)}{2}, \frac{1-\alpha}{2T_{sym}}\le|F|\le\frac{1+\alpha}{2T_{sym}} \\ 0, \frac{1+\alpha}{2T_{sym}}\le|F| \end{cases}$$

- Physically unattainable
  - The truncation affects less the spectrum
  - The pulse decreases faster than the sinc pulse
- It respects the Nyquist criterion for avoiding ISI

Raised cosine filter (2)

- The shape in time and frequency domains depends on the roll-off factor α

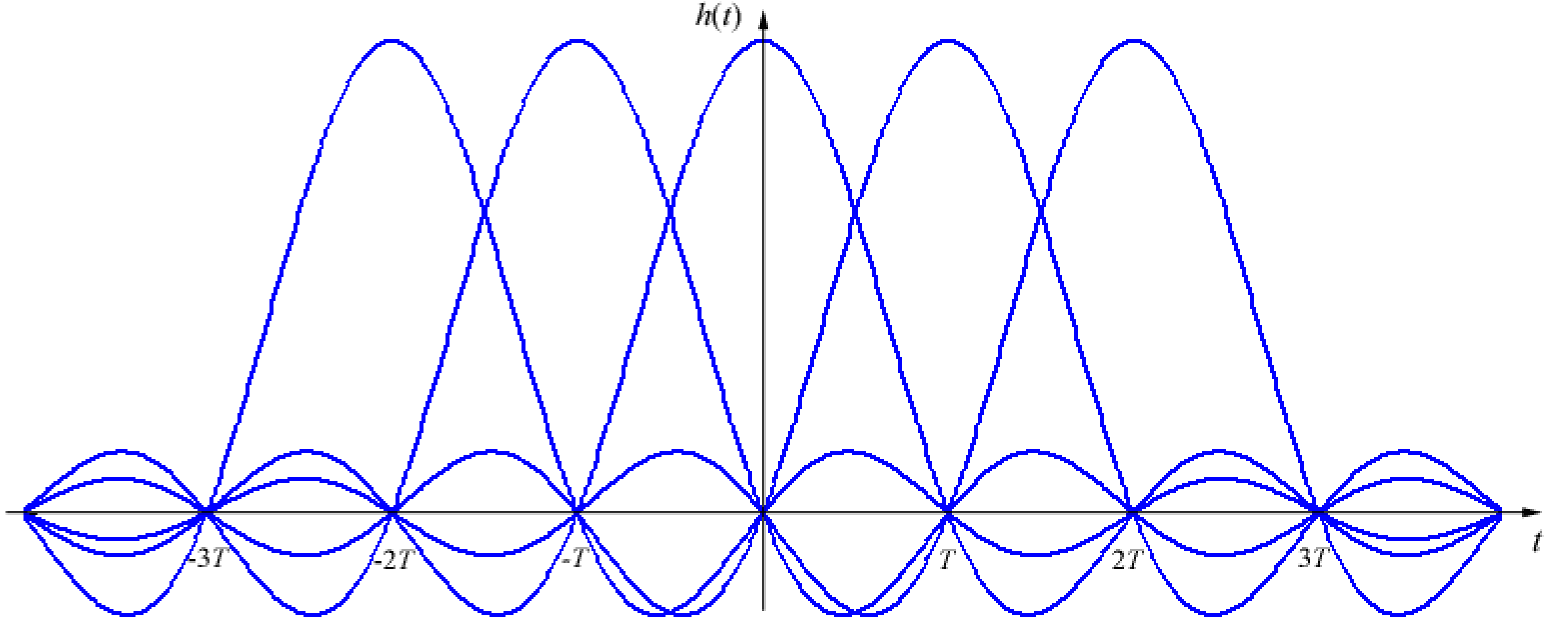




$$B = \frac{1+\alpha}{T_{sym}}$$

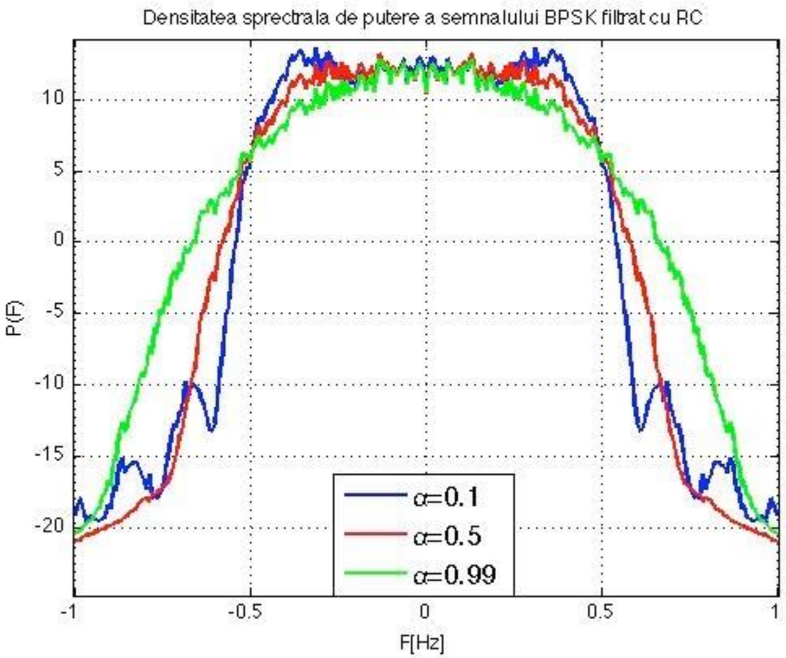
Raised cosine filter (3)

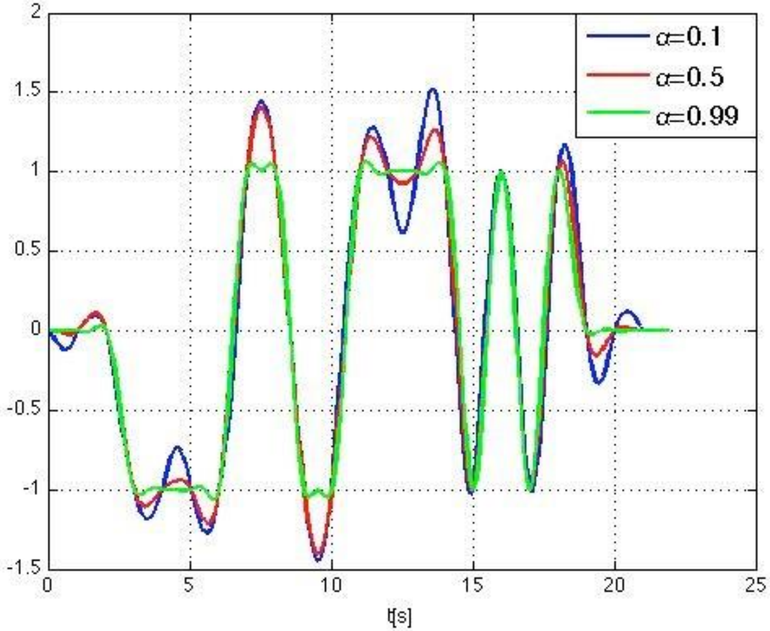
- Five consecutive symbols shaped with a RC filter



Raised cosine filter (4)

- Example: BPSK signal + pulse-shaping filter, T_{sym}=1s





Raised cosine filter (5)

- How should the roll-off factor be chosen ?
- Small α (close to 0)
  - Narrow bandwidth
  - Higher side lobes in time domain
    - Any time error results in a high ISI value
- Large α (close to 1)
  - Wider bandwidth
  - Lower side lobes in time domain
    - Small sensitivity to time errors

Root-raised cosine filter (RRC)

- Inside the receiver, the matched filter or the correlator has the same transfer function as in case of the transmitter:

$$|H_R(F)| = |H_T(F)|$$

- The global transfer function has to have a raised cosine shape, in order to avoid ISI:

$$H(f) = H_T(F)H_R(F) = H_{RC}(F)$$

- The receive and transmit filters are equal:

$$|H_T(F)| = |H_R(F)| = \sqrt{|H_{RC}(F)|}$$