3. NOISE AND NONLINEAR DISTORSIONS IN RADICOMMUNICATIONS SYSTEMS

3.1 Introduction

- Parameters that highlight the quality of the signal obtained at the output of a radio receiver are:

  - *Signal to noise ratio (SNR);*
  - *Fidelity* (for analog RR, in defining this parameter, an important role belongs to the nonlinear distortion factor);
  - *Bit error probability or bit error rate (BER)* in case of digital RR.

- The signal to noise ratio, defined at the input, accentuates the fact that at the input of the radio receiver, the useful signal is accompanied by disturbing signals:

  - *Undesired signals*
  - *Noise.*

- The majority of these perturbations are eliminated due to selectivity.

- Only a small part remains, which will influence the quality of the signal delivered to the output.

- While processing the signal, other perturbations are added: *internal noise of the RR*.

- At the same time, the signal may be processed nonlinearly, such that new components appear – *nonlinear distortion*.

- Internal perturbations represent a factor which constitutes a limit in the increase of the receiver's sensitivity.

- As it will be discussed, for analog RR, they influence the defining of the **noise-limited sensitivity.**

- In the process of measuring nonlinear distortions or noise, the two components can not be separated. This aspect will somewhat alter the results.

- For this reason, a more adequate coefficient used to appreciate the performance of a RR is the *signal to noise and distortion ratio*: **SINAD**.

- We must mention that nonlinear distortions may appear both at transmission and reception.
- In radio broadcasting, where there are a small number of radio transmitters (in comparison with the number of radio receivers), we must work in such a way that the contribution of the transmission section may be neglected.
- In other cases, the distortions which appear due to the transmission section may have a larger contribution.

- In the following, we shall start an analysis of the noise, ending by highlighting the relation that exists between the noise-limited sensitivity and the noise factor of a radio receiver.

- In the second part, we will analyze a few of the things that cause the occurrence of nonlinear distortions.

3.2 Noise and radio signal reception

3.2.1 General aspects

- As we have noticed earlier, according to the place where the perturbations that affect the signal delivered at the output are generated, we may distinguish:

    - *External perturbations*
    - *Internal perturbations*

- According to the structure, the external perturbations that accompany the useful signal at the input of a radio receiver can be:

  - *Pulse noise;*
  - *Fluctuation noise;*
  - *Radio signals.*

- If we take look at the source of the external perturbations, they may be:
  - *Industrial noise (processes);*
  - *Atmospheric noise (phenomena);*
  - *Cosmic noise (phenomena);*
  - *Noise due to the thermal agitation of some charged particles;*
  - *Noise that are coming from other radio communication systems*.
- The contribution of different types of noise depends on the operating frequency;

- A last important classification takes into consideration the frequency range in which noise occur:

1. *Noise situated in the band of the useful signal processed by the radio receiver (**in-band noise**);*

2. *Noise situated outside this band (**out-of-band noise**).*

- ***The out-of-band noise is*** mostly eliminated in the filtering process.
- The filtering function is, usually, distributed over several stages;
- A part of the mentioned noise may reach active stages that work in a nonlinear regime and they may produce undesired effects in **the band of the useful signal** (cross-modulation, interference, etc).
- Consequently, it is important to evaluate the selectivity that must be performed by the first stages, in order to obtain the performance desired in real working conditions.

- We must remark that the mentioned effects are generated inside the radio reception equipment, even though they are external from the point of view of the original cause.

- In this section, we will remind a few aspects related to the internal noise and the effect that it has over the sensitivity of the radio receiver;

- We will end this subchapter with a succint analysis of the noise coming from the antenna.

3.2.2 The internal noise of the radio receiver

3.2.2.1 The noise factor, the noise figure

- Every block of a radio receiver acts as a noise source.
- The signal and noise applied to the input are equally amplified;
- The internal noise is added to noise coming from the RR input;
- So the noise increases from input to output: the output signal to noise ratio will be smaller than the input one.

- In order to evaluate this effect, we define the ***noise factor*** F (or noise figure, NF) for a certain block:

$$F = \frac{SNR_i}{SNR_o} = \frac{P_{SI}}{P_{SO}} \bullet \frac{P_{zo}}{P_{zi}} > 1,$$

$$NF = 10\log(F) = SNR_i(dB) - SNR_o(dB)$$

- Obviously, for any block F is larger than 1. Demodulators are an exception: F may be smaller than 1.

- That is why it is convenient to define the noise factor up to the input of the demodulator.

- Finally, we shall give an alternate expression for the noise factor (more convenient for computations).

- We start from the following relation:

$$P_{so} = A_p P_{si}$$

where A_p stands for the power amplification.

- We deduce:

$$P_{zo} = P_{zi} \bullet A_p + P_{zp} \Rightarrow F = \frac{P_{zi} + \frac{P_{zp}}{A_p}}{P_{zi}} =$$

$$= \frac{P_{zi} + P_{zpi}}{P_{zi}} = 1 + \frac{P_{zpi}}{P_{zi}}$$

where $P_{zpi}=P_{zp}/A_p$ represents the overall internal noise reflected at the input of the analyzed block.

3.2.2.2 Noise sources in radio receivers

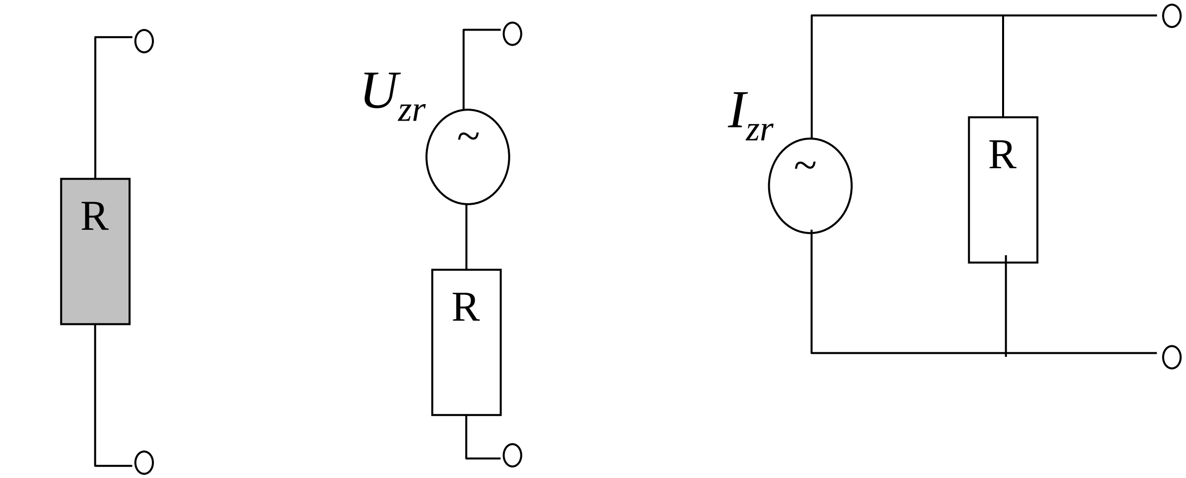
- As for any electronic equipment, the internal noise comes from:

  - Resistances
  - Selective circuits
  - Diodes
  - Transistors.

1. Resistors

- Any resistor is a source of noise;
- If we connect a sensitive enough voltmeter at its terminals, it will detect a voltage, although no current controlled by an external generator flows through the resistor.
- The voltage takes random values, both positive and negative.
- The origin of this voltage: the thermal agitation of electrons in the resistor body.

- We may use the next representation :



$$U_{zr}^{2} = 4kTBR; \qquad k = 1{,}38 \cdot 10^{-23} W / Hz \cdot K$$

$$I_{z}^{2} = \frac{4kTB}{R}$$

- In other words, a real resistor is equivalent to a noise source associated with a noise free resistor of the same value;
- The generated noise is a random signal, so it can not be characterized by its time expression or using the Fourier transform.
- It was demonstrated that the functions that we may use are: the autocorrelation function and the power spectral density (the distribution of the total power on the frequency axis).

- We may distinguish two situations:

  - *White noise;* the power spectral density is constant with frequency and has the value, let's say, N_0 for positive frequencies;

  - *Colored noise;* the power spectral density depends on the frequency.

- *The thermal noise is white noise with the power spectral density $N_o=4kT$ if the measurement is done in open circuit and $N_o=kT$ on a matched load, where T is the work temperature in K and k is Boltzman's constant;*

- In order to determine the noise voltage (or current), we must know the bandwidth and the transfer function for the equipment in which the resistor works: $H(\omega)$, respectively B_t .

- Starting from the Wiener-Hincin theorem, it was shown that for a 2-port having the transfer function $H(\omega)$ the power spectral density of the random signal resulted at the output is determined with the following relation:

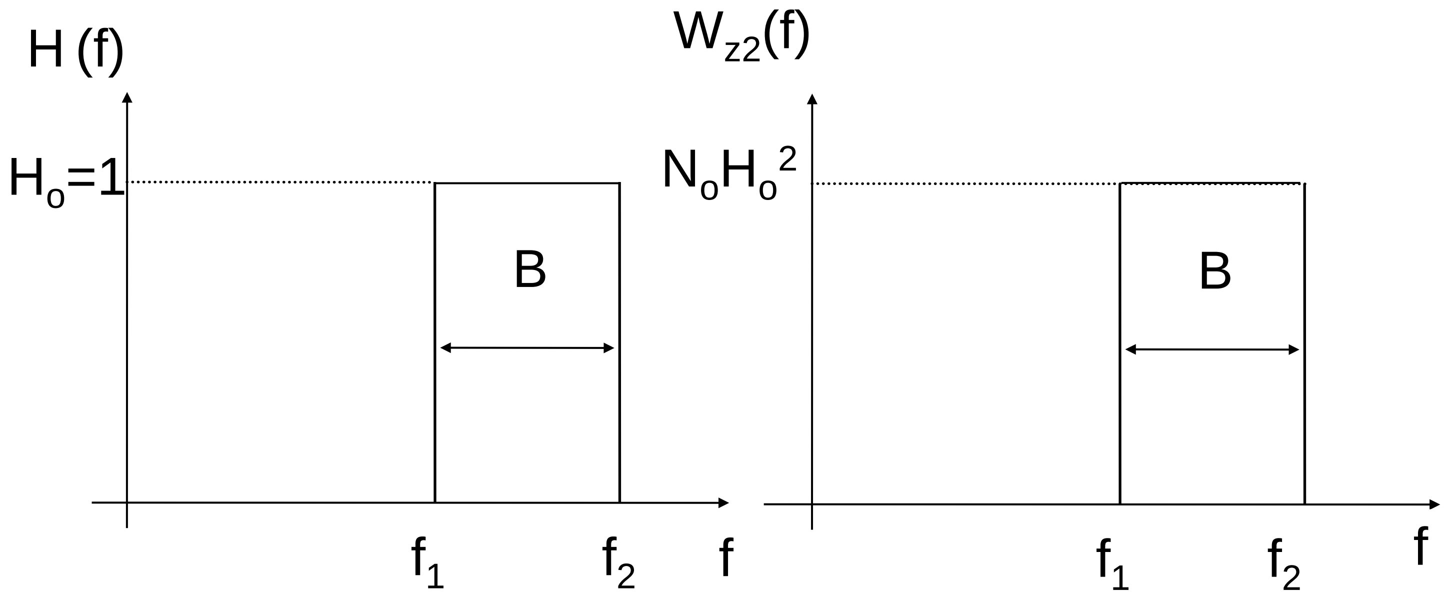
$$W_{z2}(\omega) = |H(\omega)|^2 W_{z1}(\omega)$$

- If we know the power spectral density, the power of the signal in a given bandwidth is:

$$P_z = \int_{f_1}^{f_2} W_z(f)df = \frac{1}{2\pi}\int_{\omega_1}^{\omega_2} W_z(\omega)d\omega$$

where f_1, f_2 (ω_1 and ω_2) are the limits of the operating bandwidth (we take into consideration only the domain of positive frequencies).

➢Let us take as an example an ideal band-pass filter with the following characteristic:



- The power of the output noise is:

$$P_z = N_0 B = 4kTH_o^2 B = 4kTB$$

- Taking into account the known relation:

$$P = \frac{U^2}{R} = RI^2$$

it results that:

$$U_z^2 = 4kTBR; \; I_z^2 = \frac{4kTB}{R}$$

(the Nyquist relations).

- If the filter (circuitul) is not ideal, the ***noise bandwidth, B_z*** takes the place of bandwidth B.

- The ***noise bandwidth*** *is defined as the bandwidth of an ideal filter which develops the same power at the output as a real filter, and has a gain equal to the gain of the real filter on the central frequency $H_o=H(f_o)$.*

$$P_z = H_o^2 N_o B_z = \int_o^{\infty} |H(f)|^2 \, N_0 df \Rightarrow B_z = \frac{1}{H_o} \int_o^{\infty} |H(f)|^2 \, df$$

- For example, for a CRD amplifier, we obtain:

$$B_z = 1{,}57 B_{3dB} > B_{3dB}$$

The two bandwidths get closer to each other as $k_D = B_{20dB}/B_{3dB} \to 1$.

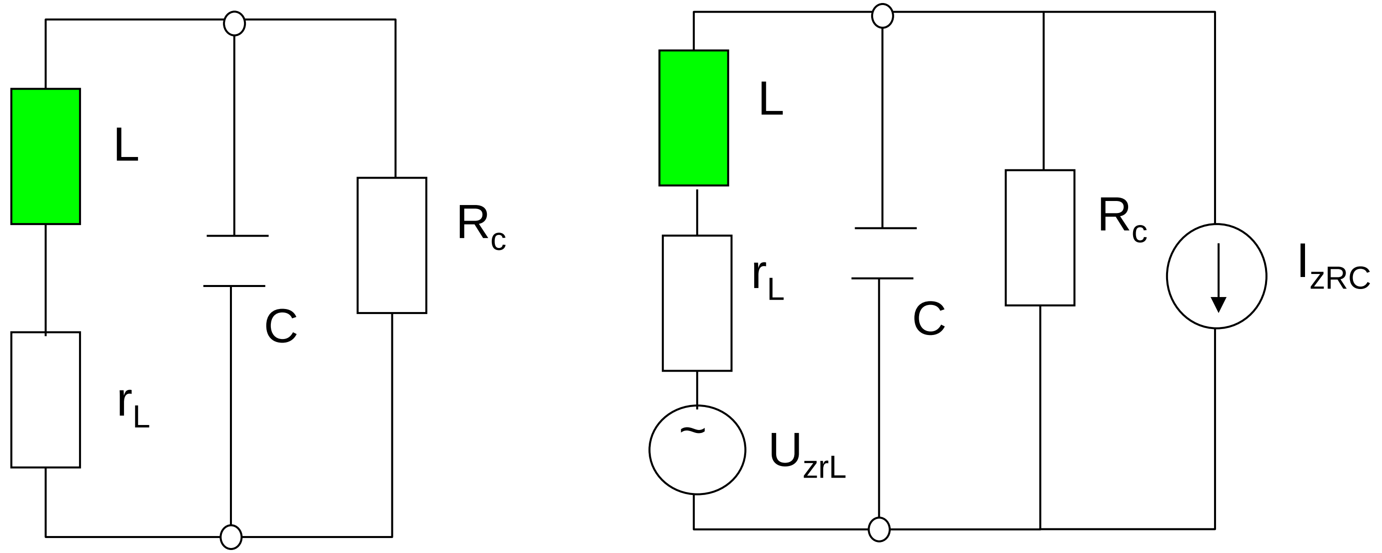
- In the end of this short presentation, we may mention that:

  - The noise of several resistors connected in a structure is equal to the noise of the equivalent resistor;

  - The importance of the resistor noise is directly proportional with the proximity of it to the input (this is true for any component), because it overlaps over a smaller signal.

2. The noise of selective circuits

- Many stages of a RR use selective circuits as load:



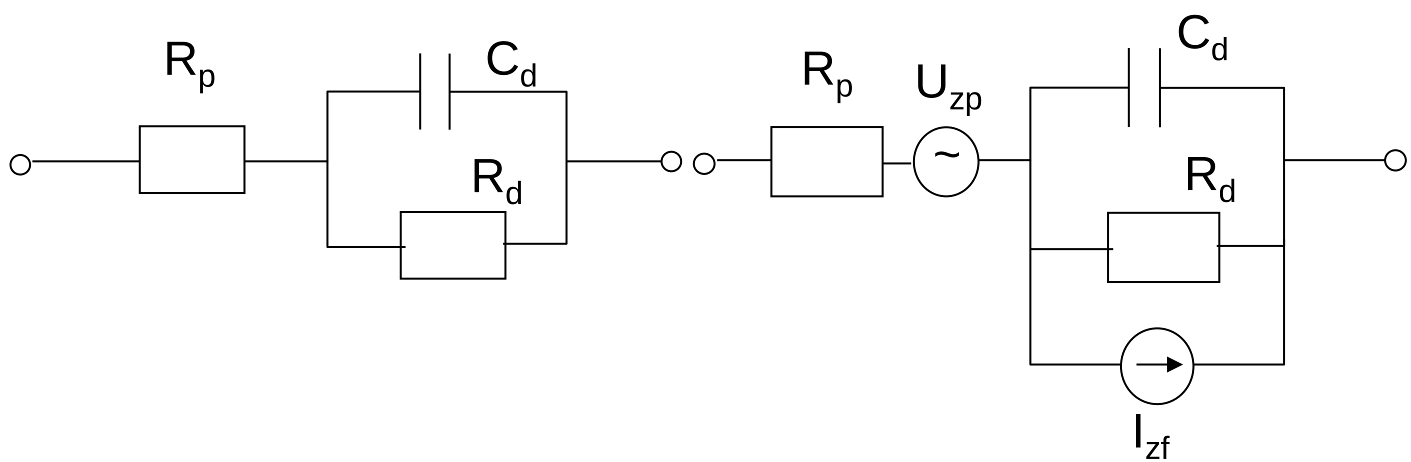
- Their noise comes from the loss resistances.

- Introducing these losses in the scheme (r_L – U_{zrL} - in series and R_c - I_{zRC} - in parallel) and going to the equivalent parallel scheme, we may prove that the total noise is equivalent to the noise generated by the equivalent parallel resistance R_d:

$$R_d = Q\omega_0 L; \quad \frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_L}; \; U_z^2(\omega) = 4kTB_z R_d$$

3 . The noise of semiconductor diodes

- they may be used in different blocks of the RR.
- from the point of view of noise effect on RR, the most important are the varicap diodes, used to tune signal circuits;
- Let us observe the equivalent scheme for a diode:



- The noise sources are characterized by:

$$U_{zp}^{2} = 4kTB_z R_p$$

$$I_{zf}^{2} = 2q(i + 2i_s)B_z$$

Here:

i – represents the diode current, i_s – the reverse current, and q – the charge of the electron.

- We are talking about white noise.

4. Noise of bipolar transistors

- Studies of the bipolar transistor have shown that the main sources of noise are :

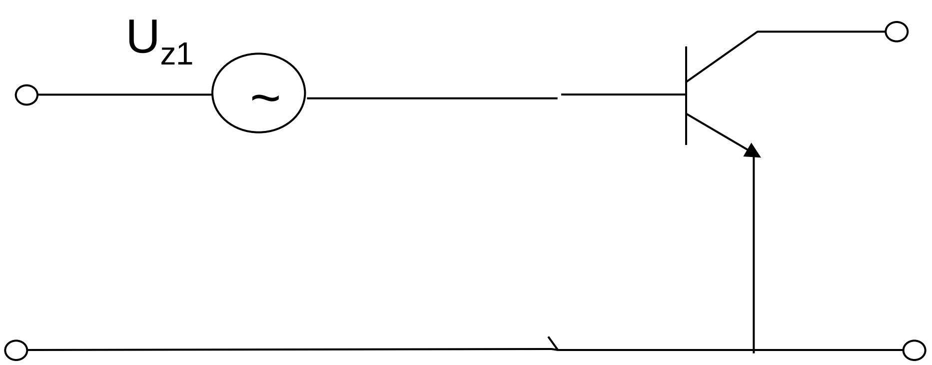
a) Thermal agitation of electrons in distributed resistances;

b) Discrete nature of electric current;

c) Capturing the charge carriers on the surface of the crystal.

a) Among the **distributed resistances**, the most important one is $R_{bb'}$. The others may be neglected in comparison to it. Its contribution may be represented by a voltage generator in series with the base:

$$U_{z1}^{2} = 4kTB_z R_{bb'}$$

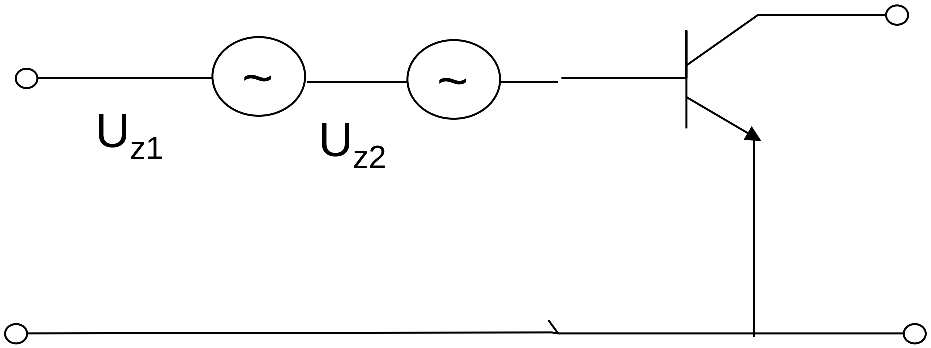


b) The noise due to the ***discrete nature of the currents*** (shot noise) may also be approximated as being white noise.

- there are two components that may not be neglected:
    - The discrete nature of the collector current;
    - The discrete nature of the base current.

- *The noise due to the collector current* may be represented as a voltage generator in series with the base:

$$U_{z2}^2 = \frac{2qB_zI_{co}}{g_m^2}$$

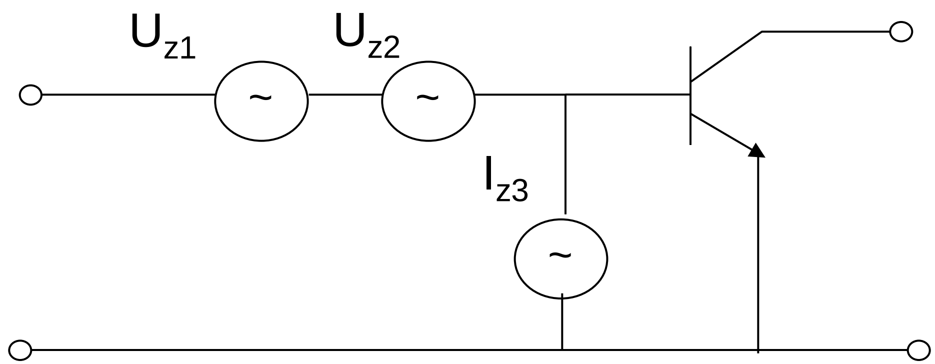


- This noise may be considered equivalent with the one generated by an equivalent noise resistance:

$$U_{z2}^2 = 4kTB_zR_{z1} = \frac{2qB_zI_{co}}{g_m^2} \Rightarrow R_{z1} = \frac{1}{2g_m}$$

- The noise coming from the base current may be represented as a voltage generator in parallel with the input:

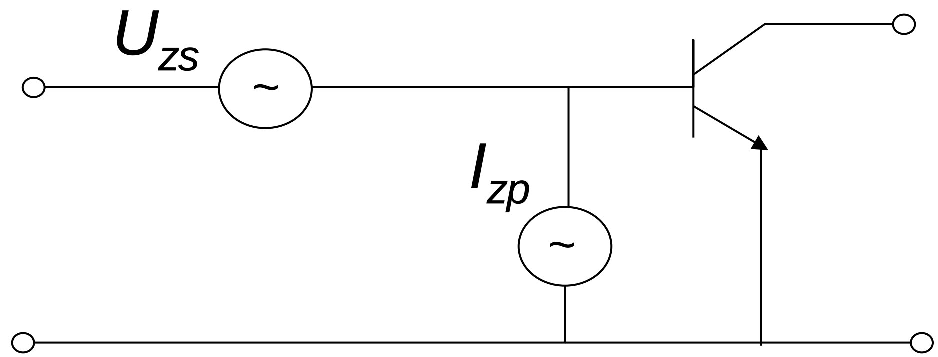
$$I_{z3}^2 = 2qI_{bo}B_z$$



- As in the previous case, we may equalize with the noise generated by an equivalent parallel resistance:

$$I_{zp}^2 = \frac{4kTB_z}{R_{zp}} \Rightarrow R_{zp} = \frac{2\beta_F}{g_m}$$

➢We may obtain the next equivalent diagram:



$$U_{zs}^2 = 4kTB_z R_{zs}; \qquad R_{zs} = R_{bb'} + \frac{1}{2g_m}$$

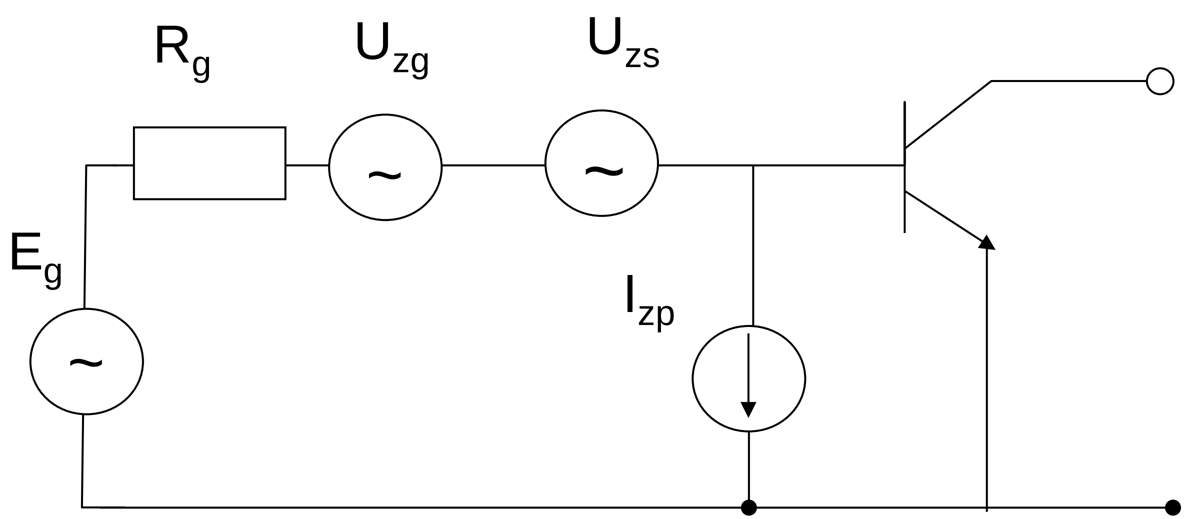
- the transistor is noiseless.

c) The process of **surface carriers capture** strongly depends on the quality of crystal processing in the neighborhood of the junction.

- After this process, a noise results, which is called *“flickering noise” or “abnormal” (noise 1/f),* and which varies inversely proportional with the frequency.

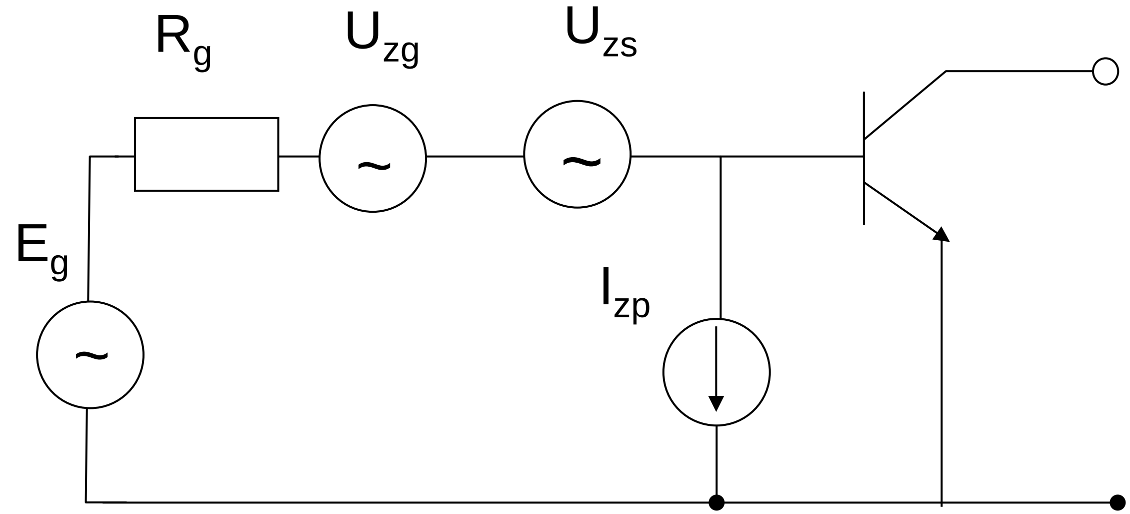
- At high frequencies, this component doesn’t count.

- Starting from the equivalent scheme shown above, we may prove that, from the point of view of behaviour in the presence of noise, there is an *optimum value of the resistance of the generator*.
- The scheme on which we define the transistor’s noise factor is the next:



$$F = \frac{P_{zit}}{P_{zg}} = \frac{U_{zit}^{2}}{U_{zg}^{2}}$$

$$U_{zg}^{\;2} = 4KTB_z R_g$$



- In order to evaluate the voltage U^2_{zit}, the current generator is equated with a voltage generator, using the Thevenin theorem:

$$U_{zp}^{2} = R_g^2 I_{zp}^{2} = \frac{4kTB_z}{R_{zp}} \bullet R_g^2$$

- We obtain:

$$U_{zit}^2 = U_{zg}^2 + U_{zs}^2 + U_{zp}^2 =$$

$$= U_{zg}^2 [1 + \frac{U_{zs}^2}{U_{zg}^2} + \frac{U_{zp}^2}{U_{zg}^2}] =$$

$$= U_{zg}^2 [1 + \frac{R_{zs}}{R_g} + \frac{R_g}{R_{zg}}]$$

$$F = 1 + \frac{R_{zs}}{R_g} + \frac{R_g}{R_{zp}}$$

- The proposed optimization leads to a minimum of the total noise voltage or of the noise factor.

- Determining the minimum of F as a function of R_g it results that:

$$R_{zo} = \sqrt{R_{zs} R_{zp}}$$

$$F_{\min} = 1 + \sqrt{\frac{1 + 2 g_m r_{bb'}}{\beta_F}}$$

- Unfortunately, it has been demonstrated that the value of R_{zo} is much different from the value corresponding to a maximum power transfer.

- For example, in an EC connection:

$$R_g(maximum\ transfer) = R_{b'e}$$

$$R_{go} = \sqrt{\frac{1}{g_m^2}(1+2R_{bb'}g_m)\beta_F} \approx \sqrt{\frac{\beta_F}{g_m}2R_{bb^i}} \approx \sqrt{2R_{bb'}R_{b'e}} \neq R_{b'e}$$

3.2.3 Evaluation of the noise introduced through the antenna

- The receiving antenna is equivalent with a voltage generator (E_a) with an internal impedance $Z_a = R_a + jX_a$.

- From the point of view of the noise, the contribution of the antenna has two components:
  - *the noise coming from the exterior, denoted with U_{zr};*
  - *the proper noise of the antenna, denoted with U_{zpa} ;*

- The latter may be considered as being *thermal noise* associated to the resistance R_a:

$$U^2{}_{zpa} = 4kT_o B_z R_a$$

- The two noise sources are independent, so their powers can be summed up:

$$U_{za}^2 = U_{zr}^2 + U_{zpa}^2$$

- By U_{za} we denote the total noise voltage from the RR input, that will affect the useful signal at the output.

- Aiming to obtain a compact expression for this voltage, we consider that it represents a fluctuation noise, thermal in nature;
- This noise is coming from the antenna resistance;
- Obviously, the antenna resistance will work at a different temperature than the real one:

$$U_{za}^2 = 4k(T_o + T_r)B_z R_a = 4kT_a B_z R_a = 4kT_o t_a B_z R_a$$

$$T_a = T_o + T_r$$

- The temperature: $$T_a = T_o + T_r = t_a T_o$$

is called *noise temperature of the antenna*, and $t_a = T_a / T_o$, *is the normalized temperature of the antenna.*

- The *normalized* temperature has been intensely studied, given the fact that it is a convenient instrument in the analysis of the real sensitivity of a radio receiver.
- It has been shown that it depends on the operating frequency and on the antenna orientation (if it is a directive one).

- For example:

  - in the range of (30...120)MHz it can be determined using the empirical expression:

$$t_a = \frac{1{,}8 \bullet 10^6}{f^3}$$

(where the frequency is given in MHz).

  - for lower frequencies(short waves), it varies a lot, reaching values of tenths.

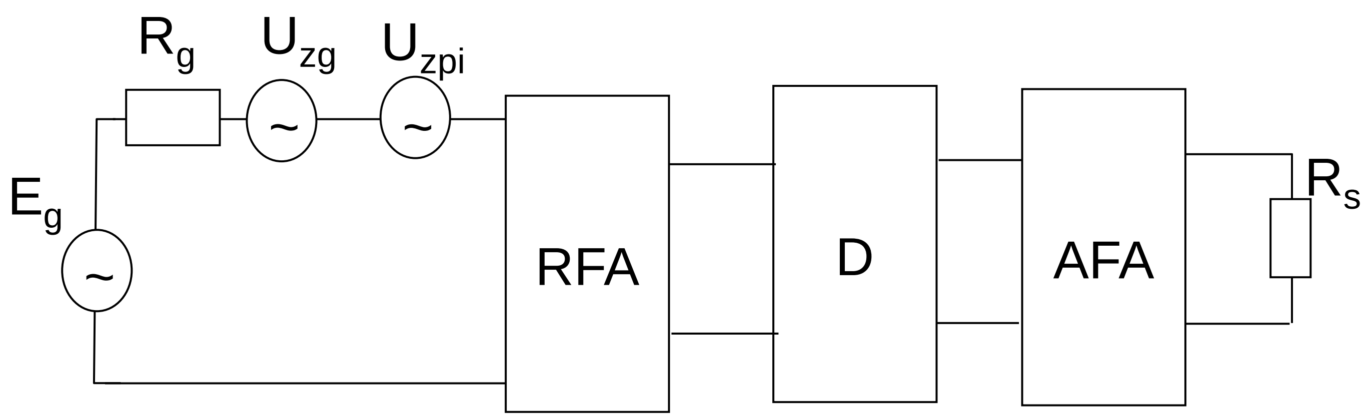
  - for values higher than 120 MHz, $t_a \approx 1$.

- Considering that the effective value of the electromotive force of the generator equivalent to the antenna is E_a we compute:

$$SNR_i = \frac{P_{SI}}{P_{ZI}} = \frac{E_a^2}{U_{za}^2}$$

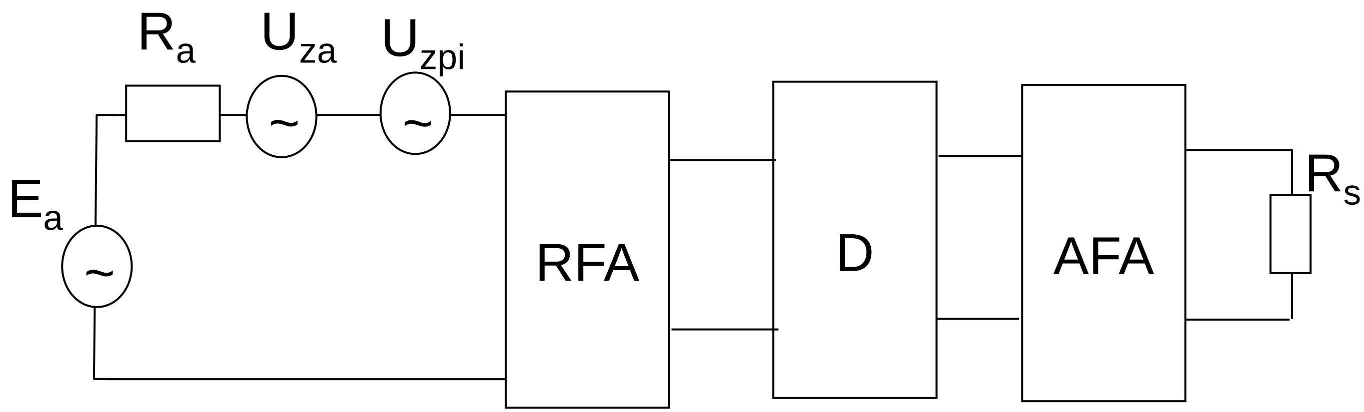
3.2.4 Evaluation of the noise limited sensitivity as a function of the noise factor

- We start from the definition of the noise factor, on the basis of the following block diagram:



$$F = 1 + \frac{P_{zpi}}{P_{zg}} = 1 + \frac{U_{zpi}^2}{U_{zg}^2} \Rightarrow U_{zpi}^2 = U^2_{zg}(F-1)$$

- Using this relation, we shall use the given diagram to evaluate the noise limited sensitivity.



- We start with the value of the SNR$_o$ at load;
- We assume that the level of the signal after the demodulator is high enough not to be noticeably affected by the noise of the low frequency amplifier which, this way, can be neglected;

- It results that:

$$SNR_{od} = SNR_o$$
$$SNR_{id} = SNR_o - \rho \text{ [dB]}$$

where ρ represents the SNR improvement upon demodulation.

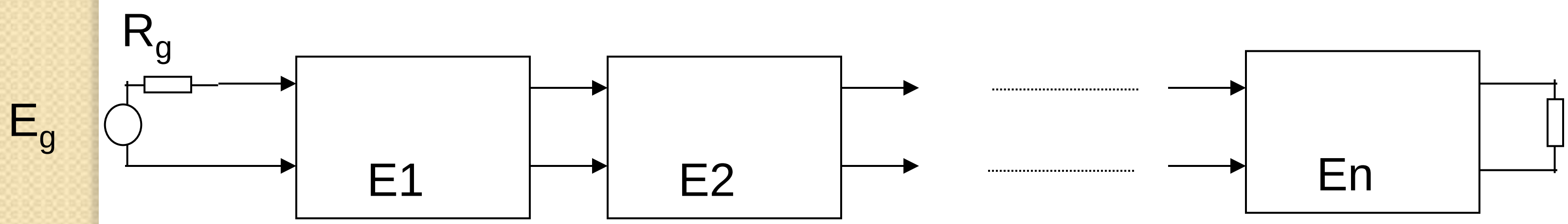
- The RFA noise has been reflected at the input, so

$$SNR_i = SNR_{id} = P_{si}/P_{zi}.$$

- The computation can be performed using either the input voltages, or the electromotive ones (the signal component and the noise component are processed similarly if the impedances may be considered resistive):

$$SNR_i = \frac{E_a^2}{U_{za}^2 + U_{zpi}^2} = SNR_{id} \quad E_a = \sqrt{4kT_oB(F-1+t_a)R_a \bullet SNR_{id}}$$

- We notice that the noise factor directly influences the noise limited sensitivity.
- *How can we intervene on it in order to control the sensitivity*?
- In order to highlight this aspect, we consider a multistage amplifier (or another similar system);
- Each stage is characterized by its power amplification, A_{pk}, and by its noise factor, F_k.



- we can prove that the global noise factor is:

$$F = F_1 + \frac{F_2 - 1}{A_{p1}} + \frac{F_3 - 1}{A_{p1} A_{p2}} + \ldots$$

- The justification of this expression is done starting from the noise factor definition:

$$F = 1 + \frac{P_{zpi}}{P_{zg}}$$

- Stages 1, 2 and 3 are characterized by:

$$P_{z1i} = (F_1 - 1)P_{zg}$$

$$P_{z2} = (F_2 - 1)P_{zg}$$

$$P_{z3} = (F_3 - 1)P_{zg}$$

- In order to transfer all noise contributions to the input, we evaluate:

$$P_{z2i} = \frac{P_{z2}}{A_{p1}} = \frac{(F_2 - 1)P_{zg}}{A_{p2}}$$

$$P_{z3i} = \frac{P_{z3}}{A_{p1}A_{p2}} = \frac{(F_3 - 1)P_{zg}}{A_{p1}A_{p2}} etc.$$

$$P_{zpi} = \sum_{1}^{n} P_{zki} = P_{z1i} + \frac{(F_2 - 1)P_{zg}}{A_{p2}} + \frac{(F_3 - 1)P_{zg}}{A_{p2}A_{p1}} + \ldots$$

$$F = 1 + \frac{(F_1 - 1)P_{zg} + \frac{(F_2 - 1)P_{zg}}{A_{p1}} + \frac{(F_3 - 1)P_{zg}}{A_{p1}A_{p2}} + \ldots}{P_{zg}} =$$

$$= F_1 + \frac{(F_2 - 1)}{A_{p1}} + \frac{(F_3 - 1)}{A_{p1}A_{p2}} + \ldots$$

- *Conclusion*: the noise factor of the first stages has the strongest effect;
- In the case of radio receivers, these stages belong to the radiofrequency amplifier.
- It is obvious that only the noise factor of the first stage will count, if it realizes a large enough gain (10-15 dB).
- Then we can approximate: $F \approx F_1$

3.2.5 Sensitivity evaluation for RR used in digital transmissions

For systems where digital modulation is used, the minimal value of the SNR_{out} (expressed as the ratio E_b/N_o) necessary for an adequate reconstruction of the desired signal is determined by the minimum bit error rate (BER) imposed by the application.

Nowadays, high performance simulators can evaluate, for a given architecture, the BER depending on noise and interferences, taking into account also the effects associated with radio propagation: fading and the attenuation caused by propagation.

Se are în vedere nivelul maxim al zgomotului cu care poate contribui receptorul și care este dat de factorul de zgomot;

This factor will be evaluated knowing the output signal to noise ratio (E_b/N_o) obtained by simulation, the sensitivity imposed by the design requirements, S_{min}, and the channel bandwidth, B.

In order to deduce the necessary expression we start with the definition of the noise factor:

$F = SNR_{in} / SNR_{out}$

- The signal to noise ratio SNR_{out} = E_b/N_o necessary for a correct detection is known;
- This corresponds to S_{int} = S_{min} (sensitivity in dBm) and N_{in} = kTB (thermic noise floor);
- As such, from the above expression, expressed in decibels, results:

$$NF = 10 \log(F) = S_{min}\,(dBm) - 10 \log(kTB) - E_b/N_o\,(dB)$$

$$S_{min}\,(dBm) = NF + 10 \log(kTB) + E_b/N_o\,(dB)$$

3.2.5 Conclusions regarding the actions that can be taken into account for noise effect reduction

- When determining the noise limited sensitivity, the following come into discussion:

  - *the proper noise of the RR;*
  - *the proper noise of the antenna;*
  - *the received noise.*

- We need to remark the fact that the last component only intervenes in determining the real work sensitivity, whereas the first two also intervene in determining the noise limited sensitivity in laboratory conditions.

- How can we act in order to ameliorate the sensitivity?

  - *In order to reduce the proper noise we shall*:

    - use low noise components;
    - identify the internal noise sources;
    - act upon the most important of them;

- *the proper noise of the antenna*:

  - in laboratory conditions, it is the noise of the equivalent generator, and it is fixed;
  - in real conditions, we may act on it, e.g. by reducing the work temperature – (measure which can be taken only for special radio communications, due to its high cost);
  - we may try reducing R_a but this is not efficient because it also reduces the useful signal.

- *the captured noise*:

  - using of directive antennas;

  - adequate placement of the antenna (industrial noise decreases with altitude).

  - use of antennas sensitive to the magnetic component of the electromagnetic wave, (this component is less disturbed).

3.3. Nonlinear distortions in radio communication systems

3.3.1 General aspects

- We shall analyze the distortions coming from the following sources:

1. *The amplitude characteristic of selective stages(filters);*
2. *The nonlinear input-output characteristic of active elements;*
3. *The interference with an RF signal;*
4. *The interference with a low frequency signal;*

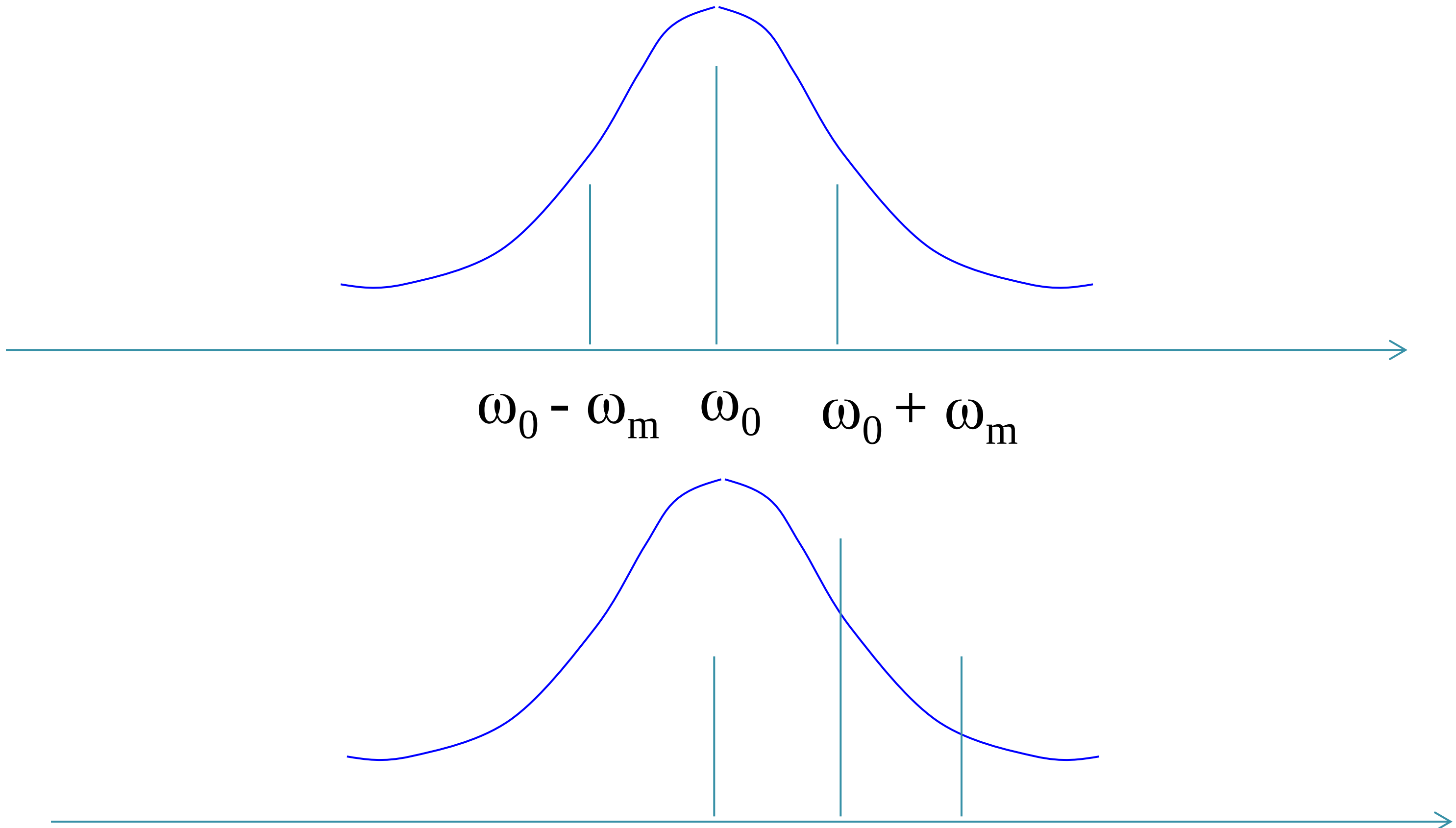
- We need to remark that the first 2 types can exist both in receivers and transmitters, whereas the last 2 are specific to radio receivers.

3.3.2 Nonlinear distortions due to the asymmetry of the selectivity characteristic

- In the following we will show that if the modulated signal is processed by means of a selective stage and its frequency characteristic is:

  - centered on the carrier frequency *only linear distortions are introduced;*

  - centered on a different frequency *both linear and nonlinear distortions are introduced.*



- Analysis will be performed for AM signals but we need to mention that the effect is similar for FM signals.

- We consider a functional block having the transfer function:

$$\underline{H}(\omega) = H(\omega)e^{j\varphi(\omega)}$$

- In a first approximation, we may neglect the effect of the phase characteristic:

$$\phi(\omega) = 0 \text{ or } \phi(\omega)=k\cdot(\omega-\omega_o).$$

$$\underline{H}(\omega) = H(\omega)$$

- The signal applied at the input is:

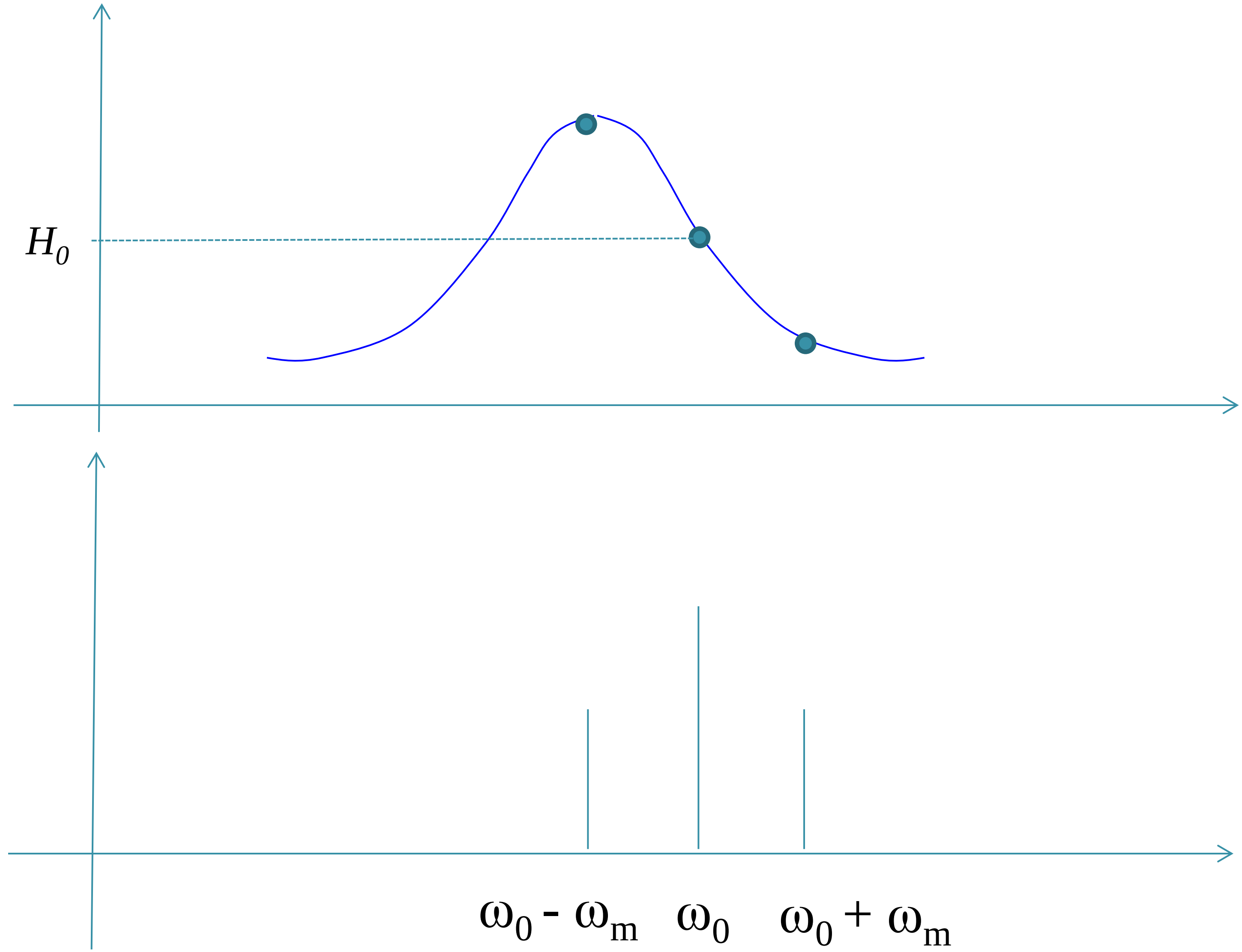
$$s(t) = U_o[1 + m\cos\omega_m t]\cos\omega_o t =$$

$$= R_e\{U_o[e^{j\omega_o t} + \frac{m}{2}e^{j(\omega_o+\omega_m)t} + \frac{m}{2}e^{j(\omega_o-\omega_m)t}]\}$$

- We will compute the answer of the filter to the complex signal, and the output signal is obtained by retaining only the real part.
- In order to simplify the expression, we introduce the notations:

$$H_o = H(\omega_o);\ H_- = H(\omega_o - \omega_m)$$

$$H_+ = H(\omega_o + \omega_m)$$


H_0
$\omega_0 - \omega_m$
ω_0
$\omega_0 + \omega_m$

- the normalized values of the transfer function:

$$h_{+} = \frac{H_{+}}{H_{o}}\,;\; h_{-} = \frac{H_{-}}{H_{o}}$$

- according to the harmonic method of computing the answer of a circuit to a signal with several components, we may write:

$$s_{2}(t) = R_{e}\left\{ H_{o}U_{o}\left[e^{j\omega_{o}t} + \frac{m\,h_{+}}{2} e^{j(\omega_{o}+\omega_{m})t} + \frac{m\,h_{-}}{2} e^{j(\omega_{o}-\omega_{m})t} \right] \right\}$$

- The analyzed signal is an AM signal, so we are interested in computing the amplitude of the resulting signal:

$$s_2(t) = S_2(t)\cos(\omega_o t)$$

$$S_2(t) = H_o U_o \sqrt{\left[1 + \frac{m}{2}(h_+ + h_-)\cos\omega_m t\right]^2 + \frac{m^2}{4}\left(h_+ - h_-\right)^2 \cdot \sin^2\omega_m t}$$

- We take a particular case for the result, for the two situations of interest:

a) Symmetric selectivity characteristic:

$$h_+ = h_- = h \Rightarrow S_2(t) = H_o U_o \left[1 + m\,h \cos \omega_m t\right]$$

$$m_2 = m\,h$$

- the degree of modulation at the output depends on the modulating frequency, so the process has introduced linear distortions.

b) Nonsymmetrical selectivity characteristic $h_+ \neq h_-$

- In order to highlight the structure of the output signal, we need to use a series expansion:

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots$$

$$\left(f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots \right)$$

- This method may be applied if the hypothesis saying that the variable term is much smaller than 1 is true; so m<<1.

$$S_2(t) = H_o U_o \sqrt{\left[1 + \frac{m}{2}(h_+ + h_-)\cos\omega_m t\right]^2 + \frac{m^2}{4}(h_+ - h_-)^2 \cdot \sin^2\omega_m t}$$

- we may write:

$$S_2(t) = H_0 U_0 [1 + m(h_+ + h_-)\cos\omega_m t + \frac{m^2}{4}(h_+ + h_-)^2 \cos^2\omega_m t +$$

$$+ \frac{m^2}{4}(h_+ - h_-)^2 \sin^2\omega_m t]^{1/2}$$

- We obtain that:

$$S_2(t) \approx H_0 U_0 \left[1 + \frac{m}{2}(h_+ + h_-)\cos\omega_m t + \frac{m^2}{8}(h_+ - h_-)^2 \sin^2\omega_m t\right]$$

- We retain from the development only the terms that lead to 2nd order distortions ($\sin^2$, $\cos^2$).

- For $h_+ = h_- = h$ we return to the preceding case.

- For the new situation, we obtain that:

$$m_2\Big|_{\omega_m} = \frac{m}{2}\left(h_+ + h_-\right)$$

$$m_2\Big|_{2\omega_m} = \frac{m^2}{16}\left(h_+ - h_-\right)^2`$$

- The demodulated signals will have amplitudes proportional to the modulation indexes; so the nonlinear distortion coefficient is:

$$d_2 = \frac{m_2\Big|_{2\omega_m}}{m_2\Big|_{\omega_m}} = \frac{m}{8}\frac{\left(h_+ - h_-\right)^2}{h_+ + h_-}$$

3.3.3 Distortions due to the nonlinear characteristic of active elements

- In general the input-output characteristic of the amplification stages, which can have a certain, nonlinear form, may be written as:

$$u_0 = a_0 + a_1 u_i + a_2 u_i^2 + a_3 u_i^3 + \dots$$

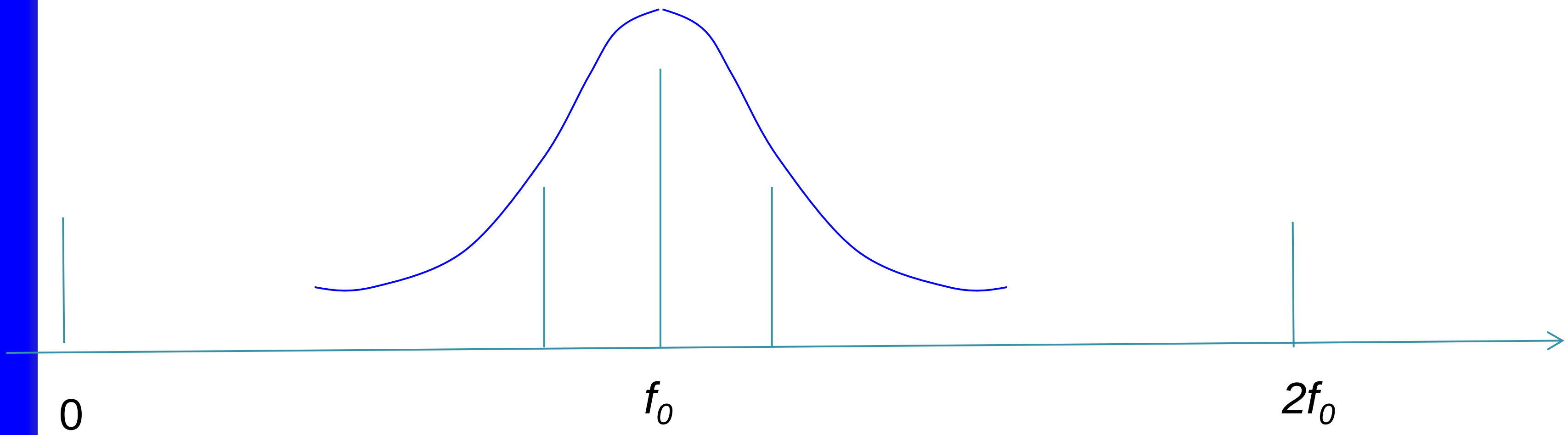
- If the input signal doesn't go over certain limits, we may approximate this expression by:

$$u_0 = a_0 + a_1 u_i$$

- We assume that this is not the case, and that the signal applied at the input is an AM signal full-carrier and a sine modulating signal (so that we can highlight nonlinear distortions):

$$u_i(t) = U_{io}\left(1 + m\cos\omega_m t\right)\cos\omega_0 t$$

- In order to rapidly simplify the expression of the signal obtained at the output, we will take into account that we are analyzing a selective amplifier having the selectivity characteristic centered on the carrier frequency f_0.



- In these conditions, we don't need to consider the components placed on $2f_0$, $3f_0$... or the DC component.

- With these observations in mind, we can write:

$$u_0^{'} = a_1 U_{io}\left(1 + m\cos\omega_m t\right)\cos\omega_0 t + a_3 U_{io}^3\left(1 + m\cos\omega_m t\right)^3 \cos^3\omega_0 t$$

- the 2nd order term generates only a low frequency component and a component around $2f_0$.

$$\cos^2\omega_0 t = \frac{1}{2} + \frac{1}{2}\cos 2\omega_0 t$$

- the 3rd order term will generate components in the range of $3f_0$ and f_0:

$$\cos^3 \omega_0 t = \frac{3\cos\omega_0 t + \cos 3\omega_0 t}{4}$$

- It results that:

$$u_0^{'} = a_1 U_{io} \cos\omega_0 t + a_1 U_{io} m \cos\omega_m t \cos\omega_0 t +$$
$$+ a_3 U_{io}^3 \frac{3}{4}\left(1 + 3m^2 \cos^2 \omega_m t + 3m\cos\omega_m t + m^3 \cos^3 \omega_m t\right)\cos\omega_0 t`$$

- In order to evaluate the component amplitudes for the signal we want to demodulate, we take into account the fact that for a AM signal, we may write:

$$u_x(t) = U_x\left(1 + m\cos\omega_m t\right)\cos\omega_0 t = U_x\cos\omega_0 t + U_x m\cos\omega_m t\cos\omega_0 t$$

$$U_d = K_d m U_x$$

*so amplitude U_d is directly proportional to the amplitude of the product $\cos\omega_m t\ \cos\omega_0 t$ which is called **modulation product.***

- Consequently, if several components appear, we evidentiate the carrier and the modulation products; the amplitude of the latter allows for computing the component amplitudes of the demodulated signal.
- For the analyzed case, if we neglect the small terms, we can write:

$$u_o^{'} = a_1 U_{io} \cos\omega_0 t + a_1 U_{io} m \cos\omega_m t \cos\omega_0 t +$$
$$+\frac{9}{8} a_3 U_{io}^3 m^2 \cos 2\omega_m t \cos\omega_0 t +$$
$$+\frac{3}{16} a_3 U_{io}^3 m^3 \cos 3\omega_m t \cos\omega_0 t$$

- It results that:

$$U_D\Big|_{\omega_m} = K_D a_1 U_{io} m$$

$$U_D\Big|_{2\omega_m} = K_D \frac{9}{8} a_3 U^3{}_{io} m^2$$

$$U_D\Big|_{3\omega_m} = K_D \frac{3}{16} a_3 U^3{}_{io} m^3$$

$$d_2 = \frac{U_D\big|_{2\omega_m}}{U_D\big|_{\omega_m}} = \frac{9}{8}\frac{a_3}{a_1}U_{io}^2 m \qquad d_3 = \frac{U_D\big|_{3\omega_m}}{U_D\big|_{\omega_m}} = \frac{3}{16}\frac{a_3}{a_1}U_{io}^2 m^2$$

- We notice that both coefficients depend on the ratio a_3/a_1 and that they can be canceled if $a_3=0$.
- So, if the signals are large, and a_3 is not 0 nonlinear distortions of modulation appear.
- The process can take place both at the transmitter and receiver.

3.3.4 Distorsions due to the interference with a radiofrequency signal

- Let there be an amplification stage in which we must consider the nonlinear terms up to order 3.

$$u_0(t) = a_0 + a_1 u_i + a_2 u_i^2 + a_3 u_i^3$$

- The signal applied at the input is:

$$u_i(t) = U_{io}\cos\omega_0 t + U_p\left(1 + m_p\cos\omega_{mp} t\right)\cos\omega_p t$$

- So, the useful signal is not modulated, and the noisy signal, considered relatively strong, is modulated.

- We will show that a modulation transfer process takes place on the carrier frequency of the useful signal.
- We take into account every observation in the preceding chapter: ω_p is a lot different from ω_0 such that the corresponding signal is eliminated by the following filters.

- Keeping only the terms around the frequency f_0 we may write:

$$u_0^{'}(t) = a_1 U_{io} \cos\omega_0 t + \frac{3}{2} a_3 U_{io} U_p^2 (\cos\omega_0 t)\left(1 + m_p \cos\omega_{mp} t\right)^2$$

- This way, modulation products of the following shape have appeared: $(\cos\omega_{mp}t)(\cos\omega_0 t)$ $(\cos 2\omega_{mp}t)(\cos\omega_0 t)$ so modulation has been transferred from the interfering signal to the useful one.

- The modulation degrees are:

$$m_{op}\Big|_{\omega_{m_p}} = \frac{3a_3U_p^2U_{io}m_p{}^2}{a_1U_{io}} = 3\frac{a_3}{a_1}U_p^2m_p = 3\frac{a_3}{a_1}U_p^2m_p$$

- So, these effects depend on the ratio a_3/a_1 and on the level of the interfering signal;

- Because the ratio a_3/a_1 has been considered in the preceding case, we may now approach a reduction of the level of this component.

$$m_{op}\Big|_{2\omega_{m_p}} = \frac{\frac{3}{4}a_3 U_p^2 U_{io} 2m_p}{a_1 U_{io}} = \frac{3}{4}\frac{a_3}{a_1}U_p^2 m_p = \frac{3}{4}\frac{a_3}{a_1}U_p^2 m_p^2$$

- This level may be reduced by the selective circuits that precede the first active stage and this is the RFTC.

3.3.5 Distorsions due to interference with a low frequency signal

- In the same conditions as in the preceding paragraph, we consider that the input signal is:

$$u_i(t) = U_0 \cos \omega_0 t + U_b \cos \omega_b t \quad with \;\; f_b < f_{m\max}$$

- We will show that if the stage operates in the nonlinear area, at the output the signal will be amplitude modulated with the low frequency signal.

- Keeping the terms from the passband of the output filter, we may write:

$$u_0^{'}(t) = a_1 U_0 \cos\omega_0 t + 2a_2 U_0 U_b \cos\omega_b t \cos\omega_0 t + \frac{3}{2} a_3 U_0 U_b^2 \cos 2\omega_b t \cos\omega_0 t$$

- Modulation products given by the signal $U_b \cos\omega_b t$ and by its second harmonic have resulted.

- We notice that in this case, the 2nd order term also has a contribution;
- The modulation degrees are:

$$m_b\big|_{\omega_p} = 2\frac{a_2 U_b}{a_1} \qquad m_b\big|_{2\omega_p} = \frac{3a_3 U_b^2}{2a_1}$$

- We notice that the only solution to decrease these terms is reducing the amplitude of the low frequency signal.

- What is the source of these perturbations?

•The noise coming from the power supply, especially if we are talking about a rectifier.

•They may come also from feedback over the power supply from the low frequency amplifier.

- The solution: a high quality filtering, that must be better towards the input stages.
- Eventually, we can use separate power supplies for the radiofrequency section and for the low frequency section.
- We can use the possibility that the transistors may work at the Q-point in which $a_3=0$;
- We may use a negative feedback etc.